MATHEMATICAL MODELS OF COGNITIVE CONTROL: DESIGN, COMPARISON, AND OPTIMIZATION

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Abstract

In this thesis, we investigate human decision making dynamics in a series of simple perceptual decision making tasks. The level of caution with which a human subject responds to stimuli is of central interest, since it influences the speed and accuracy of responses. We study the role of caution parameters in models of cognitive control processes.

We first investigate the influence of stimulus likelihood on human error dynamics in sequential two-alternative choice tasks. Errors are understood to increase in frequency when caution is low. When subjects repeatedly discriminate between two stimuli, their error rates and mean reaction times (RTs) systematically depend on prior sequences of stimuli. We analyze sequential effects on RTs, showing that relationships among prior stimulus sequences and the corresponding RTs for correct trials, error trials, and averaged over all trials are significantly influenced by the probability of alternations. Finally, we show that simple, sequential updates to the initial condition and thresholds of a pure drift diffusion model (DDM) can account for the trends in RT for correct and error trials. Our results suggest that error-based parameter adjustments are critical to modeling sequential effects. These relationships have not been captured by previous models.

In the remainder of the thesis, we compare models of human choice dynamics in tasks in which subjects must trade off between speed and accuracy in order to maximize reward rates. Caution is of critical importance: while errors decrease in frequency as caution increases, decision time increases. Direct manipulation of caution provides a framework with which to compare models. Recent work has compared the predictions of the Linear Ballistic Accumulator (LBA) and the DDM for simple RT tasks but has identified no important qualitative differences between the predictions of the two models. Comparing the fits of the two models for simple RT tasks in which subjects attempt to maximize reward rate, we show that while the pure DDM predicts a single optimal performance curve, the curve for the LBA varies significantly with model parameters. Critically, we find that while reward seeking behavior is predicted on average by an increase in caution in the DDMs, the same behavior in the best-fitting LBA model is instead predicted by a decrease in caution.

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-The Beatles

To my friends.

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Chapter 1

Introduction

The challenge of integrating teams of human and robotic agents is of growing interest. A positive consequence of the long recognized, steadily increasing capabilities of machines [76, 105, 29] is a large and growing set of ways to employ a machine to assist or even replace an individual or group in the completion of a task [129]. In order to better integrate human and robotic agents in task completion, our understanding of the human operator must keep pace with our understanding of machines.

The work presented in this thesis is motivated by the problem of understanding the behavior of the human operator in tasks shared between humans and robots. Our specific focus is upon developing a better understanding of the human operator herself while performing simple and time-sensitive tasks. In particular, given a human operator and several machines, or a combination of humans and machines, characteristic patterns of human behavior such as variations in speed and accuracy are of interest. We will focus on a special case of this human-robot system, in which one human operator must make multiple simple decisions quickly. For example, a pilot must make rapid decisions in the cockpit regarding how to turn and navigate around obstacles in her field of view [67]. Similarly, racecar drivers on the track and to a lesser extent, drivers on a public highway must deftly navigate a series of obstacles, determining the speed at which and the side from which to pass other cars [37]. In such situations, errors frequently incur a significant cost [101, 108]. In the case of military operations, the cost may be measured in lives saved or lost [131]. Systems whose evolution is governed by interactions and feedback among human and robot operators and the environment are known as human-in-the-loop (HITL) systems [36, 68].

The design of these HITL systems can be informed by the relative abilities of human and robotic agents. In general, human agents perform better at varied and interesting tasks and tend to stall or commit errors on more mundane tasks [34]. In contrast, a computer can excel at the simplest and most tedious tasks, such as sorting data or performing mathematical operations, but it will frequently struggle to process novel or unprecedented situations. However, for successful completion of several tedious tasks, such as image and text recognition in captchas, some human supervision is necessary. Conversely, certain very difficult tasks (e.g. chess [80, 24], go [13, 77], image recognition [60]) can be performed well by a computer alone when significant computing resources are allocated. Tasking the human is generally imperative but the optimal way to do so is unclear. Moreover, the design of a system with both human and robotic participants can deeply benefit from an understanding of the ways humans and robots react to environmental stimuli. Machine behavior can be well described in response to simple inputs from a human or the environment. A variety of theories and control laws [54, 117] have been developed to characterize and manipulate properties of machine behavior. Machines can be controlled, and these control laws can predict behavior even with very high levels of uncertainty in sensor measurements [7].

Significant progress has also been made towards characterizing human behavior in simple tasks. For example, speeds at which humans can detect various environmental stimuli have been well analyzed [106, 123, 35]. The quality of human visual sensing has been characterized with various psychometric curves [43, 78], and such studies have been generalized to characterize multiple animal species, including monkeys [56, 42], mice [111, 23], and rats [122]. The influence of various incentive and reward schemes on behavior has also been studied with the goal of understanding how to motivate the human to work most optimally and how to "task" the human operator. Additional factors in behavior and performance have also been considered and their roles quantified at length, including group influences [79, 118].

However, much remains unknown regarding human operator behavior, particularly regarding model development. Neurally inspired models allow us to test and compare theories and identify potential drivers of human behavior. Picking the most suitable neurally-based model is difficult. Frequently several models have been developed to describe similar data sets, and each model will have different strengths. Moreover, the models are frequently complex and incorporate multiple parameters, so a complete description of model behavior often proves elusive. Models are often highly specialized and account only for data to which they were designed to fit, and also make multiple explicit and, more problematically, implicit assumptions about the underlying biology. The best way to apply biological knowledge and modeling experience is not obvious. Multiple descriptive models exist for various aspects of behavior, for example: the firing rates of neurons and correlated relative preferences for a given decision (e.g., [126, 97, 33, 39]).

A primary goal of this work is to better model and predict human behavior in simple two choice tasks. An understanding of the influence of environmental stimuli and external feedback on human behavior will allow us to develop a deeper understanding of neuroscientific concepts, more robust mathematical models of human behavior, and finally, more efficient and useful HITL systems. To develop this understanding, we wish to use simple models as our primary tools, employing a minimum number of parameters consistent with achieving a strong predictive and descriptive power. With motivation and a toolset derived from engineering and applied mathematics, we begin a foray into some driving problems in psychology and neuroscience.

In this thesis, we consider simple binary choice tasks, in which participants must choose between two alternatives, and the decision is forced (e.g. they must select one of the discrete alternatives in order to proceed in the experiment). These are known as two alternative forced choice tasks (TAFCs). We focus primarily on the results of visual perceptual discrimination tasks. Reactions to visual stimuli have been studied at length in monkeys and humans. The passage of information can be traced from the retina through the optic nerve, via the lateral geniculate nucleus, to the visual cortex and then to the other parts of the brain, where information is integrated with non-retinal signals and a decision is made [48, 83, 53]. The accumulation process is sufficiently well-understood that neural correlates of visual discrimination TAFCs have been directly recorded from single and multiple neurons in monkeys (e.g., [81, 16, 104, 120]).

Within the visual discrimination TAFC framework, we may then address two questions of interest:

- Can we predict and model human behavior?
- How does this behavior relate to its physiological correlates?

With regard to the first question, that of predicting behavior, we consider how quickly and accurately human participants can identify environmental stimuli. The process is influenced by stimulus likelihoods as well as by reward or incentive schemes. To address the two factors, we first analyze data from two experiments which manipulate stimulus structure, and then from an experiment which incorporates performance-based rewards. We develop a simple, biologically-plausible account for human behavior in each of the tasks.

In total, we look closely at three identification and classification tasks, each requiring sequential discrimination between two discrete alternatives. The first task serves as a control in which subjects must only discriminate between each set of two alternatives presented to them, and in which the stimulus alternatives are presented in an unbiased random order. In the next task, a bias is introduced to the order of stimulus presentation. Finally, in the third of the tasks, correct and timely completion of the task is rewarded, so that subjects must attempt to trade off speed and accuracy in a manner which optimizes their rate of reward.

With regard to the second question, that of connecting physical behavior to its neural correlates, we consider a variety of modeling and physical studies. In particular, we consider adaptations of the pure drift diffusion model [47, 94, 51, 10], which has been shown to mimic aspects of neural integration. We then can refer to our model to explain trends in behavior based on both the data and the model fits.

The structure of the remainder of this chapter is as follows. First, we provide a more detailed survey of related work, including both biophysical models and the experiments which motivated them. We then identify a series of metrics used to evaluate models, and which we will use to compare our models to those presented in other work. After that, we summarize our main contributions and provide an outline of the dissertation.

1.1 Survey of Related Work

In this section, we consider the standard two alternative forced choice task (TAFC) framework, and the data collected during such an experimental task. We then describe several models in the literature which connect behavior and its neural correlates.

A typical TAFC proceeds as follows. Participation in the study begins with the administration of instructions, which may be followed by one or more training sessions. After the subject has completed these preliminary exercises, her formal participation in the experiment begins. At the beginning of each trial, a stimulus is presented to the subject. Common stimuli include visual representations such as stationary alphanumeric characters [28, 135], shapes [124, 9], or moving dots [18, 58]. After the stimulus is presented, the subject must then respond by indicating the chosen alternative, which she is generally instructed to do by pressing a button or, if eye trackers are used, initiating a saccade to a visual target. After the subject responds, a response to stimulus interval (RSI) is initiated. The RSI may be either constant or selected from a distribution of RSIs, such as an exponential distribution. After the RSI has passed, the process is repeated with the presentation of a new stimulus. In the experiments studied here, several hundred trials form a block during which experimental conditions such as stimulus probabilities remain fixed. The general procedure is illustrated in Figure 1.1.



Figure 1.1: General two alternative forced choice (TAFC) task protocol. Training and feedback are not always incorporated in a TAFC.

Behavioral data is recorded, and electrophysiological data is frequently collected as well. In particular, alongside the stimulus presented, the response indicated by the subject and her reaction time are recorded. This behavioral data can then be matched with electrophysiological correlates. In particular electroencephalogram or functional magnetic resonance imaging data for human subjects, or direct neural recordings from monkeys and other primates have proven useful in analytical and modeling efforts to tease apart the relationship: evidence is accumulated over time, in a noisy manner, and a decision is indicated when sufficient information is reached [10]. Modeling efforts take advantage of these correlations between decision making behavior and the neural recordings. We briefly summarize below a few common models. We will consider these models in greater detail in the following chapters.

The pure Drift Diffusion Model (DDM) [10] serves as a basis for much of the model construction and comparison presented in this thesis. The implementation of this model is straightforward. Noisy accumulation of evidence proceeds from a starting point x₀ at a given mean drift rate μ plus an additive Wiener process [47] with variance σ² to one of two thresholds ±z. Each of the thresholds corresponds to one of the two alternatives in the TAFC. Decision time for accumulation of evidence from some initial condition to threshold in the pure DDM is then determined by the first passage time through either threshold for the following process:

$$dx = \mu dt + \sigma dW, \quad x(0) = x_0.$$
 (1.1)

The reaction time is then the sum of the decision time defined above and the nondecision time T_0 . Nondecision time is often parameterized to represent the time required to indicate the decision after it has been made, such as the motor processes involved in a keypress, which are nonzero [10]. Nondecision time is parameterized in several models of decision making.

Given drift rate μ and noise variance σ constant, the pure DDM will make the fastest possible decision [10], and the model is therefore considered to represent an 'optimal' decision making strategy. However, this 'optimality' has a cost: pure DDM predictions lack some of the nuances of other models. For example, the pure DDM will not predict differences between RTs for error or correct trials, and it may not be able to capture individual RT distributions. To account for observed variability in RTs, additional variability in the DDM model parameters is required.

• The extended DDM [92], also known as the Ratcliff Diffusion Model, accounts for additional variability in behavior by adding variability to parameters in the pure DDM. In particular, means and variances are prescribed for the drift rate, initial condition, and nondecision time: $\mathcal{N}(\mu, s_{\mu}^2)$, $\mathcal{U}(x_0 - \frac{s_{x_0}}{2})$, $x_0 + \frac{s_{x_0}}{2}$), and $\mathcal{N}(T_0, s_{T_0}^2)$, respectively. For each trial, a drift rate μ^* and nondecision time T_0^* are chosen from normal distributions with the above properties. The initial condition x_0^* is chosen from the uniform distribution. The evolution of the extended DDM then proceeds as a pure DDM with those parameters:

$$dx = \mu^* dt + \sigma dW, \quad x(0) = x_0^*. \tag{1.2}$$

New parameters are again selected from the distributions for the following trials. The additional variation in parameter values endows the extended DDM with augmented descriptive power, and the extended model can then account for fast and slow errors as well as the corresponding overall longtail distributions characteristic of human subject RTs for correct and error trials. However, neither the pure nor the extended DDM can account for behavior in tasks with more than two alternatives. To account for multiple alternatives, a series of separate accumulators must be considered. • The Leaky Competing Accumulator (LCA) [126] allows for two or more alternatives to compete to reach a common threshold z. The mathematical setup of the pure DDM allows for only one sort of coupling in the accumulation of evidence: evidence in favor of one alternative is evidence equally against the other alternative. The LCA process allows for greater nuance, so that the accumulation processes x_i for each alternative *i* compete with all the others via the term $-b \sum_{j,j \neq i} f(x_j)$, in which $f(x_j)$ is a thresholding function. The processes also accumulate relative preference with drifts μ_i over time, and they lose excitement due to a period of inactivity (leak) $-kx_i$. The leak is proportional to relative preference for an option, and the competition is proportional to relative preference for the other options. Independent Weiner process noises W_i with variance σ^2 are again included. Decision time in the LCA is then determined by the following process:

$$dx_i = [\mu_i - kx_i - b\sum_{j \neq i} f(x_j)]dt + \sigma dW_i, \quad x_i(0) = x_{i0}.$$
 (1.3)

A constant nondecision time T_0 completes the LCA model parameterization.

• The Linear Ballistic Accumulator (LBA) [20] relaxes several of the LCA conditions, but like the LCA remains a competitive accumulation processes among the accumulators x_i . In an LBA trial, drift rates μ_i^* are selected from a normal distribution with mean μ_i and variance s, with one drift corresponding to each accumulator, as well as initial conditions x_{i0}^* drawn from a uniform distribution of initial conditions, $\mathcal{U}[0, A]$. Accumulation then proceeds linearly and 'ballistically' towards the single threshold z. The first accumulator to get to the threshold 'wins' and is selected. The accumulation

processes are then determined by the following equation:

$$x_i(t) = x_{i0}^* + \mu_i^* t, (1.4)$$

where the first x_i that reaches threshold is selected and the corresponding time at which this happens is the decision time for the task. In a small number of trials, all drift rates μ_i^* will be negative and so a decision is never made, consistent with a small portion of experimental tasks in which subjects fail to respond. A constant nondecision time T_0 completes the LBA model parameterization.

Figure 1.2 illustrates key features of each of the models described above.

1.2 Model Comparison Metrics

We desire numerical and systematic means by which to compare the mathematical models we have considered. We wish to identify models with descriptive capability but relatively few parameters. We therefore focus on two published metrics designed to penalize extra parameters and reward descriptive capability in the models: the Bayesian Information Criterion (BIC) [107] and the Akaike Information Criterion (AIC) [3]. Criterion are defined so that a lower value corresponds to a better model fit.

The BIC attempts to maximize the likelihood that a given set of parameters describes a data set. BIC is defined by

$$BIC = n \ln\left(\frac{RSS}{n}\right) + k \ln(n), \qquad (1.5)$$

in which RSS is the residual sum of squares for the model predictions. The parameters n and k are the sample size or number of points the model is designed to fit and the number of parameters in the model, respectively.

The AIC was inspired by the BIC and was designed with similar goals in mind. The AIC value can be calculated using the following expression:

$$AIC = n \ln\left(\frac{RSS}{n}\right) + 2k.$$
(1.6)

A corrected value of AIC, the AIC_c can be used, and this corrected value is recommended for small k or large n [21], such as in small data sets and complex model designs.

$$AIC_{C} = n \ln\left(\frac{RSS}{n}\right) + 2k + 2k \cdot \frac{k+1}{n-k-1}.$$
(1.7)

1.3 Main Contributions of Dissertation

This dissertation contributes to the literature on sequential effects, post-error slowing, and optimal performance and the speed-accuracy tradeoff. The work presented in the second and third chapters on sequential effects has previously appeared in its entirety in a journal [52]; an earlier version of work presented in the fourth chapter has been presented in a poster at the Psychonomics Society Annual Meeting but appears here in full for the first time. A journal version of this material is also currently in preparation. The main contributions of this dissertation are as follows.

- We developed a new model which accounts for both post-error slowing and sequential effects. Our adapted DDM is based on a pure DDM with systematic variation of the initial condition and thresholds, driven by responses on previous trials.
- 2. We conducted an original experimental study showing that sequential effects vary with probability of alternations. We also showed that our adapted DDM could account for these sequential variations in RT and ER with the probability of alternations.
- 3. We identified key differences between the LBA and DDM accounts of behavior in reward maximization tasks. In particular, we compared fits to experimental data (kindly provided by Fuat Balci [6]) and analytical predictions for the behavior of both high- and low-earning subjects. We found that while both the DDM and LBA models could adequately predict behavior in choice tasks, the two models gave very different explanations for average subject behavior: the DDMs predicted that participants exercised more caution as the tasks became easier, whereas the best-fit LBA predicted that they exercised less caution.

To do this, we focused on the role of the caution parameter in a series of neurally-plausible models. For the first two contributions, we adjusted caution differently after correct and error trials. For the final contribution, we allowed caution to be manipulated by overall preferences for relative speed or accuracy.

1.4 Outline of Dissertation

The structure of this dissertation is as follows. The next two chapters consider sequential patterns in RT. The fourth chapter investigates the influence of reward on overall RT and ER, and hence on the speed-accuracy tradeoff.

In Chapter 2, we consider sequential effects in simple, unbiased RT tasks. We separately consider RTs for correct trials, error trials, and on average, as well as the error rates. Comparing these data points with those predicted by prior model fits to a data set, we find poor fits. We develop a new model, adapted from the pure DDM, and we show that this model recreates sequential effects for error and correct trials, whereas the other models do not.

In Chapter 3, we consider the results of an original experiment in which sequential effects are studied in response to biased random stimuli, for which either repetition or alternation trials are more likely. Applying our model from the previous chapter, we again find that it accounts for trends in the data.

In Chapter 4, we compare the pure and extended DDM accounts of simple choice behavior with the accounts of the LBA describing performance in reward maximization tasks. We show that while each model can account for average subject behavior, and both DDMs can also account for the highest earning subjects, the LBA accounts for behavior by predicting participants behave with less caution in instances in which the DDM predicted that they used greater caution.

The final chapter details our conclusions and directions for future work.



Figure 1.2: Comparison of popular models of the decision making process: the Drift Diffusion Model (DDM), extended DDM, Leaky Competing Accumulator Model, and Linear Ballistic Accumulator Model.

Chapter 2

Responses to Unbiased Random Stimuli^{*}

Efforts to model and predict human behavior are informed by an understanding of the dynamics of error rates (ERs) and reaction times (RTs) in simple tasks. In particular, in two-alternative forced-choice (TAFC) tasks (e.g., [72, 73, 74, 96]) human participants are known to slow down after committing an error and generally to exhibit RTs and ERs that systematically depend on prior stimulus sequences [8, 25, 72, 102, 69, 130, 114, 113]. However, while much previous work has considered post-error slowing and sequential effects separately, we are not aware of studies that explicitly account for interactions among these effects. In this chapter we consider the effect of post-error slowing on sequential RT patterns in tasks in which subjects are responding to unbiased random stimuli.

^{*}This chapter is presented with approximately 90% of the text and figures extracted verbatim from parts of [52]. Exceptions include minor textual changes throughout and extended details of the experimental setup, which are new.

Patterns in RTs for individual trials are well documented in the literature. In particular, relative to their mean RTs on correct trials, subjects are known to respond faster on error trials and more slowly immediately following errors [89, 71, 70]. On average it has been shown that participants return to their mean RT values within two trials after an error [91]. Various models of TAFC tasks have accounted for this post-error slowing [96, 40]. In addition, RTs and ERs are known to vary systematically with repeating (R, current stimulus is the same as the previous stimulus) and alternating (A, present stimulus differs from the previous stimulus) stimuli even when stimulus order is selected randomly and each stimulus is equally likely [8, 71, 114]. Several other TAFC models account for these sequential effects [28, 66, 46]. However, to our knowledge the mean RTs on trials following specific sequences of stimuli have not been studied independently for trials ending in an error, and deliberate post-error adjustments have not been incorporated into models of sequential effects.

In this chapter, we study sequential patterns in ERs as well as in RTs for error and correct responses independently in TAFC tasks in which stimuli are equally probable with no bias towards either repetitions or alternations, focusing on sequences of three trials. We reanalyze behavioral data from an equal-probability experiment [28] with a relatively long response to stimulus interval (RSI, 800 ms). In the following chapter, we will consider responses to stimuli with constant bias towards repetitions or alternations.

To further study patterns in RT and ER we extend the pure drift diffusion model (DDM) to account for sequential patterns. As noted in Chapter 1, Section 1.1, the pure DDM describes choice between two alternatives by representing the noisy accumulation of the difference in evidence (logarithmic likelihood) from a given initial condition to one of two decision thresholds. This process is known to mimic aspects of neural integration [26, 50, 10, 49]. Adapting the DDM, we propose two simple update mechanisms to vary the initial condition and thresholds from trial to trial, depending on previous stimuli and response correctness. We show how our adapted DDM can account for the observed trends in RT for correct and error trials.

Related TAFC models frequently involve a variant of the leaky competing accumulator (LCA) [126], featuring two coupled stochastic differential equations which contain multiple parameters to account for leakage (decay of previous evidence) and for the interaction between neural populations. LCA models have been shown to capture sequential effects for equally-probable stimuli [28, 46]. For certain parameter ranges, it can be shown that the LCA, along with race, inhibition, and other models, reduces to a DDM [10], and the DDM itself may be extended to account for variability in the model parameters [97]. However, we are aware only of modeling studies that predict both ERs and RTs for sequential effects [28, 46], and these studies did not analyze patterns in error RTs, nor did they incorporate post-error parameter adjustments into the analysis. Bayesian models of TAFC, which can also be represented by DDMs for certain parameter ranges [75], have also been used to model sequential effects [133, 132], but none of these models yet accounts for patterns in errors.

Physiological evidence suggests sources of systematic changes in behavior from trial to trial, providing some neurobiological basis for our proposed update mechanisms. An electroencephalogram (EEG) study has identified a SE pattern in the P300 response [116], an event related potential signal which follows 300-600 ms after unexpected, alternating, stimuli. The prefrontal cortex is also activated following an alternation after frequent repetitions, with greater activation following a longer run of repetitions prior to the alternation [57]. In addition, the anterior cingulate cortex (ACC) is known to show increased activity with increased conflict in representation, or alternation of stimuli, and ACC activity has been linked to cognitive control and post-error corrections and corresponding increase in RT [12]. Prior work has incorporated ACC conflict signals into models of sequential and error effects [66].

This chapter is organized as follows. In Section 2, we describe the experimental protocol, first reported in [28]. We then describe a diffusion model account of participant behavior. In Section 3, we describe the experimental results and discuss diffusion model fits to participant behavior. Finally, Section 4 contains further discussion and our conclusions, and identifies directions for future experimental and modeling work. Mathematical details are relegated to an Appendix.

2.1 Materials and methods

In this section, we describe the protocol followed for the experiment presented in this chapter. We then describe a general model of decision making, which accounts for choice behavior with two simple mechanistic adaptations to the pure drift diffusion model (DDM). Finally, we describe a procedure for fitting the model to match participant data in our adapted DDM.

2.1.1 Experiment 1: Unbiased random stimuli

In the experiment (reanalyzed from [28]), subjects participated in a classic two alternative forced choice task. Seated in front of a computer screen, with their hands on a keyboard, the subjects were presented with a series of stimuli in sequence. After the subject identified the current stimulus with a keypress, the next stimulus would then be presented after a short delay period. Stimulus probabilities were equal and transition probabilities were held constant at 50%. As the details of the experiment have been described in the literature previously, we outline them only briefly here.

Six Princeton University undergraduates participated in a task over a single session by identifying the upper or lowercase "o" character on the screen with the appropriate keypress. The index finger was used to identify the uppercase letter, and the middle finger to identify the lowercase letter. Each session consisted of 13 blocks of 120 trials each, and a response to stimulus interval (RSI) of 800 ms was used. Participants received course credit in exchange for their participation in the study: correct responses were not specifically rewarded, nor were errors penalized. For additional details see [28]. No trials were omitted from our reanalysis.

2.1.2 An adapted drift diffusion model

To account for sequential and error effects, we consider a simple adaptation of the pure drift diffusion model (DDM) [96, 97, 10] in which the initial condition and thresholds are updated sequentially following each trial. In the pure DDM, information is accumulated stochastically according to the following equation:

$$dx = \mu dt + \sigma dW, \quad x(0) = x_0. \tag{2.1}$$

Here x(t) represents the difference in logarithmic likelihood ratio for the two choices, the drift rate μ (conventionally taken to be positive) represents the difference in incoming evidence for the correct alternative relative to the incorrect alternative, and σdW is a Wiener (white noise) process with mean 0 and variance σ^2 . The evidence thresholds are set at $\pm z$, and noisy accumulation continues until x(t) first crosses either +z (a correct decision) or -z (an error). If the non-decision time is given by $T_{\rm nd}$ such that RT = DT + $T_{\rm nd}$ where DT is the decision time, it can be shown that the mean DT and ER are [47, 22]:

$$\langle \text{DT} \rangle = \tilde{z} \tanh(\tilde{z}\tilde{\mu}) + \left\{ \frac{2\tilde{z}(1 - \exp(-2\tilde{x}_0\tilde{\mu}))}{\exp(2\tilde{z}\tilde{\mu}) - \exp(-2\tilde{z}\tilde{\mu})} - \tilde{x}_0 \right\}, \quad (2.2)$$

and

$$\langle \text{ER} \rangle = \frac{1}{1 + \exp(2\tilde{z}\tilde{\mu})} - \left\{ \frac{1 - \exp(-2\tilde{x}_0\tilde{\mu})}{\exp(2\tilde{z}\tilde{\mu}) - \exp(-2\tilde{z}\tilde{\mu})} \right\},$$
(2.3)

in which the parameters have been scaled so that

$$\tilde{z} = \frac{z}{\mu}, \quad \tilde{x}_0 = \frac{x_0}{\mu}, \text{ and } \quad \tilde{\mu} = \left(\frac{\mu}{\sigma}\right)^2.$$
 (2.4)

Given a nonzero initial condition \tilde{x}_0 , mean DTs are different for correct and error trials:

$$\langle \mathrm{DT}_{\mathrm{correct}} \rangle = \frac{\exp\left(\left(\tilde{z} - \tilde{x}_{0}\right)\tilde{\mu}\right)}{1 - \mathrm{ER}} \\ \times \frac{\left(\tilde{z} - \tilde{x}_{0}\right)\cosh\left(\left(\tilde{z} + \tilde{x}_{0}\right)\tilde{\mu}\right)\sinh\left(2\tilde{z}\tilde{\mu}\right) - 2\tilde{z}\sinh\left(\left(\tilde{z} - \tilde{x}_{0}\right)\tilde{\mu}\right)}{\sinh^{2}\left(2\tilde{z}\tilde{\mu}\right)}, \quad (2.5)$$

$$\langle \mathrm{DT}_{\mathrm{error}} \rangle = \frac{\exp\left(-\left(\tilde{z} + \tilde{x}_{0}\right)\tilde{\mu}\right)}{\mathrm{ER}} \\ \times \frac{\left(\tilde{z} + \tilde{x}_{0}\right)\cosh\left(\left(\tilde{z} - \tilde{x}_{0}\right)\tilde{\mu}\right)\sinh\left(2\tilde{z}\tilde{\mu}\right) - 2\tilde{z}\sinh\left(\left(\tilde{z} + \tilde{x}_{0}\right)\tilde{\mu}\right)}{\sinh^{2}\left(2\tilde{z}\tilde{\mu}\right)}. \quad (2.6)$$
See the Appendix for derivations of Eqs. (2.5-2.6).

The simplicity and analytical tractability of the DDM is a motivating factor in our decision to use it as a basis for our study. We note that the DDM is much simpler than the Leaky Competing Accumulator (LCA) Model [126], which has been used in prior models of sequential effects [28, 66, 46]. LCA processes involve two or more coupled nonlinear and stochastic differential equations (see Chapter 1, section 1.1 above). We shall compare predictions of the adapted DDM with the LCA-based Cho [28], Jones [66], and Gao [46] models in Section 2.2, using the data of Experiment 1.

Priming mechanism

As with other sequential effects models (e.g., [28, 66, 46]), parameters are updated by a priming mechanism to reflect the stimulus history of repetitions and alternations and its influence on subject behavior. In the Cho, Jones, and Gao Models, priming is implemented by small history-based changes to the drift parameter, μ . In contrast, in our adapted DDM we update the initial conditions at trial n + 1by setting

$$\tilde{x}_0(n+1) = \pm k \left(M(n) - \frac{1}{2} \right) \pm \tilde{x}_{\text{offset}},$$
(2.7)

in which n is the previous trial number, k > 0 is a scaling constant, and M(n) serves as a dynamic memory of repetitions and updates at the start of each new trial. M(n) is confined to the interval [0,1], so that $M(n) - \frac{1}{2}$ ranges from $-\frac{1}{2}$ to $\frac{1}{2}$. A symmetry between R and A biases is then enforced: a positive value of $M(n) - \frac{1}{2}$ corresponds to bias towards R trials and a negative $M(n) - \frac{1}{2}$ corresponds to bias

towards A trials. Moreover, updates to M(n) are defined such that an increase in bias towards R trials will correspond to a decrease in bias towards A trials, and vice versa. Without loss of generality, we define our model terms such that the positive direction for $\tilde{x}_{\text{offset}}$ always corresponds to the correct response. The normalized drift parameter $\tilde{\mu}$ must then always take a positive value, and the sign of the offset bias $\tilde{x}_{\text{offset}}$ and the scaling constant k will vary from trial to trial, with positive coefficients selected if the current trial is a repetition of the previous stimulus and negative coefficients if it is an alternation.

The memory function is updated as follows:

$$M(n) = \Delta M(n-1) + \begin{cases} 1-\Delta, & \text{if repetition from } n-1 \text{ to } n, \\ 0, & \text{if alternation from } n-1 \text{ to } n, \end{cases}$$
(2.8)

where $0 < \Delta < 1$. The parameter Δ determines the dependence of behavior on previous trials, with higher values corresponding to the level of influence of trials further back in the sequence and lower values corresponding to dependence on only recent trials. A Δ value of 0.5 corresponds to a memory length of approximately four trials ($\Delta^4 = 0.0625$), after which history dependence drops below 5 percent. A single update parameter Δ can then account for responses to both R and A trials. In contrast, the Cho, Jones, and Gao models [28, 66, 46] used a memory function M(n) but separately tracked R and A trials. Our model is always initialized with no bias, so that $M(1) = M(2) = \frac{1}{2}$, after which M(n) updates according to the above expression. This mechanism allows for large adjustments to initial conditions to follow the termination of strings of repetitions or alternations. The updating mechanism is similar to updates to biasing terms proposed in previous work [28, 46], in which initial conditions and drift rates are updated.

Error-correcting mechanism

We also employ error-correction threshold modulation. Threshold modulation has been studied in the context of several sequential choice tasks [10, 109]. In particular, models have used variable thresholds in describing optimal behavior, as well as to account for variability in reaction time. Increased caution is attributed to a higher threshold, which is understood to follow error commission. However, prior models of sequential effects have not included threshold modulation.

In the adapted DDM, the thresholds are adjusted after every trial and constrained to remain symmetric at $\pm \tilde{z}$. After a correct trial, \tilde{z} is reduced by $\tilde{z}_{\text{down}} > 0$, and after an error trial, increased by $\tilde{z}_{\text{up}} > 0$:

$$\tilde{z}(n) = \tilde{z}(n-1) + \begin{cases} -\tilde{z}_{\text{down}}, & \text{if correct at } n-1, \\ \tilde{z}_{\text{up}} & \text{if error at } n-1. \end{cases}$$
(2.9)

The range of $\tilde{z}(n)$ is constrained so that the thresholds always have a magnitude greater than or equal to the magnitude of the initial conditions, i.e., such that $\tilde{z}(n) \geq \frac{k}{2} + \tilde{x}_{\text{offset}}$; $\tilde{z}(n)$ is also constrained so that $\tilde{z}(n) \leq \tilde{z}_{max}$. The thresholds are initialized conservatively such that $\tilde{z}(1) = \tilde{z}_{max}$. If an update causes $\tilde{z}(n)$ to fall outside its bounds, $\tilde{z}(n)$ is then set to the value of the nearest bound until the next trial.

Sequential, error-correcting variations in the evidence thresholds $\tilde{z}(n)$ can produce significant differences between reaction times for correct and error trials. Trials with lower thresholds have higher ERs and faster RTs; thus, on average, error trials are faster and correct trials slower. This effect is modulated by adjustments to the initial condition \tilde{x}_0 , which result in faster correct or incorrect responses by biasing the system asymmetrically to start nearer one of the thresholds. The memory function and initial condition and threshold updates remove \tilde{z} , and add six parameters to the model: k, $\tilde{x}_{\text{offset}}$, Δ , \tilde{z}_{down} , \tilde{z}_{up} , and \tilde{z}_{max} , in addition to $\tilde{\mu}$ and T_{nd} , for a total of eight parameters.

2.1.3 Model simulation and data fitting procedure

Fitted model parameters were used to validate the adapted DDM against data from Experiment 1. The data were sorted by sequence, RT, and ER. Model behavior was computed for each parameter set and then sorted similarly. The model was run using the same stimulus sequences that each participant had encountered. Parameters were selected by attempting to minimize the sum of squared errors between model prediction and participant data,

$$Err = \sum_{i=1}^{N} (r_{i,\text{model}} - r_{i,\text{data}})^2, \qquad (2.10)$$

in which the elements r_i include unweighted overall mean RTs for each of the four possible second-order sequences for R and A stimuli. We considered RR, AR, RA, and AA (R followed by R, A followed by R, etc.) sequences for correct trials, for error trials, and for trials overall, mean ERs for these sequences, as well as mean RTs before error trials, on error trials, and after error trials. For the experiment described in this chapter, r had N = 19 elements. Time was measured in units of seconds and ERs in decimal fractions of trials, so that ranges of values for elements of r were comparable.

The search for parameters was conducted using a Trust-Region-Reflective Optimization (TRRO) algorithm [31, 32]. The function lsqnonlin in Matlab was used with default options to search and select parameters that minimize Eq. (2.10). For each parameter set and experimental condition, the model ran at least 5 times through the stimulus sequence that each participant had encountered in a given block of trials. (Thus, if a participant were to see big, then big, then small "o" stimuli, the model was presented with those same stimuli in sequence big-big-small, along with the stimuli preceding and following them, and these entire sequences would be repeated for the model subject at least 5 times.) For each trial the probability of error was computed from Eq. (2.3) and from this number the correctness or error of that trial was decided by biased coin flip. The expected correct or error RT for the trial was then obtained from Eq. (2.5) or Eq. (2.6), and parameter updates were implemented according to Eqs. (2.7-2.9). The individual trial results were then sorted and averaged in the same manner as the experimental data, model predictions were inserted into Eq. (2.10), and model parameters were updated by the TRRO algorithm. This was repeated until the lsqnonlin convergence criterion was met. The model was then simulated, with the converged paramter values, being run 10 times on the stimulus sequences that each participant had encountered, to produce the averaged model results displayed below.

Use of the analytical expressions of Eqs. (2.3-2.6) for expected ERs and RTs substantially speeds up the fitting process, since direct numerical simulations of Eq. (2.1) are avoided. The final parameter selections are listed in Table 1, and the results and implications of the fitting process are considered in the results and discussion sections of this chapter.

Table 2.1: DDM Parameterization for Experiment 1 of [28]

$ ilde{\mu}$	$T_{\rm nd}$	k	$\tilde{x}_{0,\text{offset}}$	Δ	$\tilde{z}_{ m down}$	$ ilde{z}_{ m up}$	\tilde{z}_{\max}
38.1747	0.2626	0.0943	0.0051	0.6860	0.0058	0.0348	0.2857

2.2 Results

In order to better understand the relationship between sequential and error effects, data from the experiment was sorted by stimulus sequence and response correctness and compared with model predictions. We first note several trends from this analysis in the data. We then validate our model fit by comparing it with the data from Experiment 1.

In our analysis, we refer to RA and AR sequences as *unexpected sequences*, and RR and AA sequences as *expected sequences*. The RT for an RA sequence is the RT corresponding to the A trial, and for an AR sequence, the RT corresponding to the R trial. We call an R line one which connects plotted data for RR and AR, and an A line one which connects plotted data for RA and AA. We consider only the two most recent trials in each sequence in our calculations, as the effects of errors are known to persist only for a limited duration [89].

We consider sequential effects and error effects in data from Experiment 1 [28] (referred to as Cho Data), in which R and A trials were equally likely, and as has been customary, we initially average over all responses, correct and incorrect. We first discuss overall sequential effects in RT and ER, as shown in Figure 2.1. As

expected, we find the smallest mean RT and ER for expected trials (RR, AA), and the largest mean RT and ER for unexpected trials (AR, RA). The effects of sequence on RT (F(3, 15) = 14.81, p < 0.001, $\eta^2 = 0.13$) and ER (F(3, 15) = 8.80, p < 0.01, $\eta^2 = 0.25$) were significant in two one-way, within-groups ANOVAs, statistical tests designed to compute the likelihood that means of different groups of variables are equal [30].

We consider also three published, generative models of the data in the experiment, which we refer to as the Cho [28], Jones [66], and Gao [46] Models, respectively. These models were designed to account for these basic sequential effects, and we note that they, as well as the adapted Drift Diffusion Model (DDM) described in Section 2.3, account for trends in mean RTs and ERs.



Figure 2.1: Mean (a) RTs and (b) ERs for the data in the Experiment 1 (Cho Data), the three published fits to the data (Cho, Jones, Gao), and the fit presented for this data in the adapted DDM of the present study. In the diagram, an RR sequence refers to the RT on the second repetition of a stimulus (e.g. left, left, left) and an AR sequence refers to the RT on the first repetition of a stimulus (e.g. left, right), etc. The adapted DDM provides the best fit to RTs, but underestimates errors, especially for AA sequences.

We next consider the data separated into correct and error trials, shown in Figure 2.2(a) as solid and dotted lines, respectively. Splitting the data in this way reveals a separation in mean RT for correct and error responses that is greatest for unexpected trials (AR, RA) and least for expected ones (RR, AA). For unexpected trials, error responses are fast, and correct responses are slow. A two-way within-groups ANOVA shows that the effect of response correctness is significant $(F(1,5) = 113.93, p < 0.001, \eta^2 = 0.35)$, along with the interaction of response correctness and expectedness of a stimulus $(F(1,5) = 16.51, p < 0.01, \eta^2 = 0.19)$. We note a slight asymmetry in the responses such that RTs for error and correct trials are closer for the R lines than for the A lines. Figures 2.2(b,c) illustrate the results of the Cho and Jones Models, respectively, and Figures 2.2(d,e) those of the two versions of the Gao Model. While all four of these models capture the trends in RT for correct trials, none of them predicts the qualitative patterns or quantitative results for error trials. Since the ERs are generally low, RTs averaged over both correct and error trials are close to the mean RTs for the correct trials alone, and this failure of the models becomes apparent only when error trials are considered separately (cf. Figure 2.1, which displays fairly good fits, and see [28, 66, 46]). This analysis also reveals that the errors, while fast on average, are not uniformly so, being significantly faster for unexpected sequences (AR, RA). Moreover, as shown in Figure 2.2(f), the adapted DDM accounts for all the RT data.

Strikingly, we note that when plotted against each other as in Figure 2.3, RTs for correct and error trials for the sequences RR, AR, RA, and AA display a monotonic and nearly linear relationship, which we call the *sequential RT tradeoff*. As we shall see in the next chapter, such a tradeoff also holds for data from



Figure 2.2: Sequential effects from Experiment 1: (a) data, (b) results of the Cho Model [28], (c) results of the Jones Model [66], (d-e) results of the two Gao Models [46], and (f) results of the adapted DDM. The Cho and Jones models predict a dimensionless reaction time, which we give here in nondimensional units (ndu, b, c). The adapted DDM captures the slopes of the R and A curves for error and correct trials. In these figures, the correctness or lack thereof of a given trial corresponds only to that trial itself, so a left-right-left sequence is tabulated as correct for the final left stimulus if and only if the final trial was identified correctly as a left stimulus.

Experiment 2 (Chapter 3). In Figure 2.3 we show the data from Experiment 1, which is described in this chapter ($R^2 = 0.995$, p < 0.01) and the adapted DDM, and from a separate study by Jentzsch and Sommer [65] ($R^2 = 0.96$, p < 0.05), which both have strong correlations and high values of the correlation coefficient R [30]. The area of the circles are proportional to the ERs for the given sequences. We note that the smallest ERs correspond to sequences with relatively fast correct responses and slow errors, while the high ERs occur with relatively fast errors and slow correct responses. While the overall ordering of the sequences (RR, AR, RA, AA) in the tradeoff differs between the two experimental studies, in both cases the points corresponding to unexpected trials (AR, RA) lie at the upper left, and those corresponding to expected trials (RR, AA) lie at the lower right.



Figure 2.3: Sequential RT tradeoff for unbiased tasks: a slower RT for correct trials corresponds to a faster RT for error trials for the sequences RR, AR, RA, and AA. The RT tradeoff for Experiment 1 is shown in red. Also shown, in blue: the RT tradeoff from a prior study by Jentzsch and Sommer [65]. Adapted DDM fits to data from Experiment 1 are shown in black. The areas of the circles are proportional to the ERs. The smallest and largest ERs are approximately 2% and 10%, respectively.

The ordering of the tradeoffs is influenced by the nature of the task. However, in each task we see that an increase in time to respond correctly (or a bias towards the correct response) is correlated with a decrease in time to respond in error, and vice versa. Our proposed biasing mechanism achieves a similar effect.

Finally, we consider the RTs before, during, and after an error in Experiment 1, as shown in Figure 2.4. Mean RTs for trials immediately following an error are longer than both those for the error trial itself and for the trial immediately before the error. A one-way within-groups ANOVA confirms that this effect on RT is significant (F(2, 10) = 16.37, p < 0.001, $\eta^2 = 0.48$). We again compare the behavior with the adapted DDM and the three previous models. In the Cho Model, the RT after an error is slower than the RT on the error trial but faster than the trial immediately prior to the error. The Jones Model maintains the trends in the data but parameter values are skewed so that the range of RTs is larger. In the two Gao Models, mean RTs for trials immediately preceding and following an error are faster than those on the error trial itself: opposite to the data. The adapted DDM provides the best fit, with the RTs for error trials and post-error trials closely matching the data, although it underestimates RTs on the pre-error trial.

We compare the adapted DDM with the other models using the Akaike Information Criterion (AIC, [1, 119]), corrected AIC (AIC_c, [59, 21]), and Bayesian Information Criterion (BIC, [2, 112], which provide model fit comparisons that account for the number of parameters included in each model, as described in Chapter 1 Section 1.1. We also compute the square of the correlation coefficient R [103], which quantifies the predictive relationship between between actual and predicted values of experimental data. Scores for the different model fits are shown in Table 2. The adapted DDM receives the best (lowest) overall and relative scores on all three metrics, confirming the fit qualities shown in Figures 2.1-2.4. AIC_c values cannot be computed for the Gao model, because the number of means being compared is too close to the number of parameters used in the model itself.

Model	Total Parameters	R^2	AIC	AIC _c	BIC
Adapted DDM	8	0.996	84.7	115.1	108.2
Cho	13	0.936	138.2	237.0	176.5
Gao 1	18	0.915	140.4	-	193.4
Gao 2	18	0.951	137.0	-	190.0
Jones	16	0.943	146.9	450.9	194.1

Table 2.2: Model Peformance Comparison



Figure 2.4: Post-error slowing in data from Experiment 1 and in the models of Cho [28], Jones [66], Gao [46], and the adapted DDM of the present study. In data from Experiment 1, the mean RT for a trial immediately following the error trial is slower than that for the trial before the error, and the mean RT for the error trial itself is fast. The Cho model [28] captures the latter qualitatively but exaggerates it, and the Gao models [46] fail to account for both trends. The Jones Model accounts for the proper trends but overestimates the magnitude of post-error slowing. The adapted DDM accounts for both trends but underestimates post-error RTs.

2.3 Discussion

In this chapter, we propose priming and error-correcting mechanisms to account for sequential effects and post-error slowing, respectively. Each mechanism, on its own, is commonplace in models of decision making. Indeed, various priming mechanisms have been previously proposed to account for sequential effects [66, 28, 46]. Post-error slowing is also known to occur and exert a significant influence on RT patterns [89, 90, 71]. The implementation of post-error slowing is understood to be a simple one: in an accumulator model, the response thresholds can be raised following an error to increase the necessary processing time before a decision is reached [87, 88, 15, 61]. However, to the best of our knowledge, no prior model of sequential effects has explicitly incorporated such an error-correcting mechanism to also account for post-error slowing.

Our model is informed by previous work: the initial conditions are varied according to a priming function similar to those in other models [28, 66, 46], and the thresholds are raised after incorrect responses and lowered after correct ones [109]. Variability in thresholds of drift diffusion processes during a trial can result in fast errors [96]. Our implementation, however, is unique: we use both priming and error-correcting mechanisms in the same model. In doing so, we can account for many of the observed trends in behavior.

Our adaptation of the pure drift diffusion model has multiple advantages. The pure DDM is analytically simple, and explicit expressions exist for both RT distributions and accuracy, and separate and closed form expressions for mean RTs can be derived for correct and error responses, as shown in the Appendix of this chapter. With nonzero initial conditions, the pure DDM can also account for RT distributions for correct and error trials. Moreover, the priming and errorcorrection mechanisms that we have proposed are conceptually straightforward. With the error-correction mechanism, our model accounts for post-error slowing: the RT for the trial which immediately follows an error trial is not only significantly slower than the error trial but also slower than the RT for the trial immediately preceding the error. We show that when thresholds are systematically adjusted to account for error and correct responses and priming is implemented, sequential patterns in error and correct response trial RTs emerge and are consistent with participant behavior, as shown in Figure 2.4.

In the following chapter, we will see that the adapted DDM also accounts for RT patterns when R and A trials are not equally likely. There we will discuss in greater detail how the design of the adapted DDM allows us to capture this behavior.

2.4 Appendix: Derivation of RTs for Correct and Error Trials

In this section, we derive the mean reaction time for the drift diffusion model (DDM) conditioned on hitting either the upper z_u or lower $-z_l$ boundaries, and for a general initial condition $x_0 \in (-z_l, z_u)$.

Suppose that x(t) is the position of a Brownian particle at time t. The dynamics of the movement of this particle are governed by the drift diffusion equation:

$$dx = \mu dt + \sigma dW \tag{2.11}$$

$$x(0) = x_0, (2.12)$$

in which μ is the deterministic drift of the particle, x_0 is the starting position, and σdW are independent white noise (Weiner) increments of variance σ^2 . We assume that the particle is allowed to move until it hits either an upper boundary $x(T) = z_u$ or a lower boundary $x(T) = -z_1$ where T is the hitting time. In this case, the joint densities of the hitting time for boundaries at z_u and $-z_1$ are given by

$$g(t, x(T) = z_{\rm u}) = \frac{\pi \sigma^2}{(z_{\rm u} + z_{\rm l})^2} e^{\frac{\mu}{\sigma^2}(z_{\rm u} - x_0)} \sum_{n=1}^{\infty} n e^{-\alpha_n t} \sin\left(\frac{n\pi(z_{\rm u} - x_0)}{z_{\rm u} + z_{\rm l}}\right), \quad (2.13)$$

$$g(t, x(T) = -z_{\rm l}) = \frac{\pi\sigma^2}{(z_{\rm u} + z_{\rm l})^2} e^{-\frac{\mu}{\sigma^2}(z_{\rm l} + x_0)} \sum_{n=1}^{\infty} n e^{-\alpha_n t} \sin\left(\frac{n\pi(z_{\rm l} + x_0)}{z_{\rm u} + z_{\rm l}}\right), \quad (2.14)$$

where $\alpha_n = \frac{1}{2} \left[\frac{\mu^2}{\sigma^2} + \left(\frac{n\pi\sigma}{z_u + z_l} \right)^2 \right]$ and $t \ge 0$ (cf. [41, 92, 97]).

To obtain the conditional densities, one must divide the above equations by the probability of hitting that particular boundary, i.e. $g(t|x(T) = z_u) = \frac{g(t,x(T)=z_u)}{P[x(T)=z_u]}$; these latter probabilities are [41]

$$P[x(T) = z_{\rm u}] = \frac{e^{-2\frac{\mu x_0}{\sigma^2}} - e^{2\frac{\mu z_1}{\sigma^2}}}{e^{-2\frac{\mu z_{\rm u}}{\sigma^2}} - e^{2\frac{\mu z_1}{\sigma^2}}},$$
(2.15)

$$P[x(T) = -z_{\rm l}] = \frac{e^{-2\frac{\mu z_{\rm u}}{\sigma^2}} - e^{-2\frac{\mu z_{\rm l}}{\sigma^2}}}{e^{-2\frac{\mu z_{\rm u}}{\sigma^2}} - e^{2\frac{\mu z_{\rm l}}{\sigma^2}}}.$$
(2.16)

Thus, the mean reaction time conditioned on hitting the upper boundary is given by

$$\langle T \rangle_{|z_{\mathbf{u}}} = \int_{0}^{\infty} tg(t|x(T) = z_{\mathbf{u}})dt$$

$$= \frac{1}{P[x(T) = z_{\mathbf{u}}]} \frac{\pi\sigma^{2}}{(z_{\mathbf{u}} + z_{\mathbf{l}})^{2}} e^{\frac{\mu}{\sigma^{2}}(z_{\mathbf{u}} - x_{0})} \sum_{n=1}^{\infty} \frac{n\sin\left(\frac{n\pi(z_{\mathbf{u}} - x_{0})}{z_{\mathbf{u}} + z_{\mathbf{l}}}\right)}{\alpha_{n}^{2}}.$$
(2.17)

Fortunately, a closed-form expression exists for the sum of the infinite series in the previous equation [86, 125]:

$$\sum_{n=1}^{\infty} \frac{n\sin(ny)}{(C^2 + D^2 n^2)^2} = \frac{1}{D^2} \left[\frac{\pi y}{4\frac{C}{D}} \frac{\cosh\left((\pi - y)\frac{C}{D}\right)}{\sinh\left(\pi\frac{C}{D}\right)} - \frac{\pi^2}{4\frac{C}{D}} \frac{\sinh\left(y\frac{C}{D}\right)}{\sinh^2\left(\pi\frac{C}{D}\right)} \right].$$
 (2.18)

Setting $C^2 = \frac{\mu^2}{2\sigma^2}$, $D^2 = \frac{(\pi\sigma)^2}{2(z_u+z_1)^2}$ and $y = (z_u - x_0)$, after some algebra, we arrive at a closed form for the mean decision time conditioned on hitting the upper boundary:

$$\langle T \rangle_{|z_{\mathbf{u}}} = \frac{1}{P[x(T) = z_{\mathbf{u}}]} \frac{1}{\mu} e^{\frac{\mu(z_{\mathbf{u}} - x_{0})}{\sigma^{2}}} \\ \times \frac{(z_{\mathbf{u}} - x_{0}) \cosh\left(\frac{\mu(z_{1} + x_{0})}{\sigma^{2}}\right) \sinh\left(\frac{\mu(z_{\mathbf{u}} + z_{1})}{\sigma^{2}}\right) - (z_{\mathbf{u}} + z_{1}) \sinh\left(\frac{\mu(z_{\mathbf{u}} - x_{0})}{\sigma^{2}}\right)}{\sinh^{2}\left(\frac{\mu(z_{\mathbf{u}} + z_{1})}{\sigma^{2}}\right)}.$$
(2.19)

In a similar fashion we obtain the mean decision time conditioned on hitting the lower boundary:

$$\langle T \rangle_{|-z_1} = \frac{1}{P[x(T) = -z_1]} \frac{1}{\mu} e^{\frac{-\mu(z_1 + x_0)}{\sigma^2}} \\ \times \frac{(z_1 + x_0) \cosh\left(\frac{\mu(z_u - x_0)}{\sigma^2}\right) \sinh\left(\frac{\mu(z_u + z_1)}{\sigma^2}\right) - (z_u + z_1) \sinh\left(\frac{\mu(z_1 + x_0)}{\sigma^2}\right)}{\sinh^2\left(\frac{\mu(z_u + z_1)}{\sigma^2}\right)}.$$
(2.20)

Chapter 3

Responses to Biased Random Stimuli^{*}

Characteristic patterns in speed and accuracy following sequences of repetitions and alternations are well documented only for tasks in which the stimuli are equally likely. While overall trends in speed and accuracy have received significant attention [26, 97, 10, 110], for tasks in which the stimuli are not equally likely, such sequential patterns in mean RT have not been considered. In this chapter, we consider sequential effects in RT in tasks in which the order of stimulus presentation is randomly biased towards either repetitions or alternations.

In a majority of TAFC studies, participants are either rewarded equally for overall participation or they are rewarded for each correct response. However, several studies (e.g. [33, 42]) have investigated tasks which reward correct responses to one stimulus more highly than another and have shown that reward contin-

^{*}This chapter is presented with approximately 90% of the text and figures extracted verbatim from parts of [52]. Exceptions include details of the experimental setup, which are new.

gencies influence choice behavior. When reward probabilities or reward values are unequal, participants are known to select the stimulus corresponding to the most probable or most valuable reward more frequently [33, 42], and they may do so almost optimally [42].

When stimuli are equally probable and correct responses are equally rewarded, several stimulus sequence effects are known. For small (< 500 ms) response to stimulus intervals (RSIs), the behavior typically illustrates automatic facilitation (AF): mean RTs on the current trial are faster if the previous trial was a repetition, regardless of whether the current trial is a repetition or an alternation. For slow RSIs (\approx 1000 ms), mean RTs on the current trial after a series of alternations are faster if the current trial is an alternation and slower if the current trial is an repetition [8, 72, 69]. This effect is called strategic expectancy (SE), suggesting a relationship between a participant's expectations and his or her reaction time. Moreover, a transition occurs from AF to SE as RSI increases [113, 65] and can be illustrated graphically [4]. Prior to the present work, it was unknown whether such a transition from AF to SE could also occur for a constant RSI with increasing probability of alternations, or, more generally, how sequential effects carry over from equally-probable to biased stimuli.

In this chapter, we study sequential patterns in ERs as well as in RTs for error and correct responses independently in TAFC tasks in which stimuli are equally probable or strongly biased towards repetitions or alternations, focusing on sequences of three trials. Stimulus sequences can be biased by specifying stimulus probabilites (state orientation) or by specifying transition probabilities between states (transition orientation), and it is known that these processes produce distinct response patterns in RT [19]. Since we are interested in sequential effects and expectancy, we generate stimuli by a first-order Markov process with unequal (as well as equal) transition probabilities (see Figure 3.1): The biased case sets the probability of an alternation (P_A) to be unequal to the probability of a repetition $(1 - P_A)$. Transition probabilities P_A and $1 - P_A$ are held fixed over blocks of trials, and we use relatively long RSIs (800 ms and 1,000 ms mean), for which SE is most apparent. In this chapter, we collect and analyze a new data set with P_A set to 10%, 50%, and 90%. We find significant transition probability effects on RTs for error and correct responses and on ERs.

An understanding of the relationship between error correction and sequential biasing mechanisms may allow us to further differentiate between corresponding physiological processes. Such an understanding could have broad implications. Indeed, recent work suggests that the same mechanisms that account for sequential effects also account for sequence learning [115]: a general mechanism may therefore lend insight into sequence learning [113, 84, 85, 44], including linguistic processes. Further, better understanding of the mechanisms behind simple discrimination tasks may also allow for improved prediction and prevention of errors.

This chapter is organized as follows. In Section 2, we describe Experiment 2. We then apply the diffusion model account of participant behavior developed in Chapter 2, Section 2.1.2 to describe the behavior of human subjects in this task. In Section 3, we describe the experimental results and discuss diffusion model fits to participant behavior. Finally, Section 4 contains further discussion and our conclusions, and identifies directions for future experimental and modeling work.



Figure 3.1: Stimulus order is generated by a transition-oriented Markov process. Given current stimulus 1, the next stimulus will be stimulus 2 (an alternation) with probability P_A and stimulus 1 (a repetition) with probability $1 - P_A$.

3.1 Materials and methods

In this section, we describe the protocol followed for Experiment 2. We then describe the adapted DDM model fits, performed as in Chapter 2, Section 2.1.3, and we compare the resulting parameters with those obtained in Chapter 2. We also provide an interpretation of the fitted parameter values.

3.1.1 Experiment 2: Biased Random Stimuli

In this experiment stimulus transition probabilities were varied from block to block, so that in a given block a participant would have a constant high, medium, or low probability of alternations. That is, given the current stimulus 1, a participant would next see stimulus 2 with probability P_A and the same stimulus 1 again with probability $1 - P_A$, with the sequence of stimuli being drawn from a transitionoriented Markov process, with P_A fixed during each block, as shown in Figure 3.1.

Participants

Sixteen adults (6 males) participated in exchange for a standard payment of \$12 per session of 9 blocks. Participants were recruited from the Princeton University community via announcements posted online and on campus. The experiment was approved by the Institutional Review Panel for Human Subjects of Princeton University, and all participants provided their informed consent prior to participation.

Stimuli

Participants performed an RT version of a motion discrimination task. The visual stimulus consisted of a black screen showing a cloud of white moving dots with a red, stationary fixation dot at its center. The red dot had size 0.30 degrees visual angle, and the white dots had size 0.15 degrees each and moved within a circle of diameter 10 degrees at a speed of 7 degrees/second and a density of 20 dots/degree². On each trial 90 percent of the white dots would move coherently in a given, "correct" direction, and the remaining white dots would move randomly. The high coherence of motion was selected to ensure that some processing was necessary but that the difficulty of the task would remain low, consistent with other studies of sequential effects. A decision could be indicated with a left or right keypress at any point after dots appeared on the screen. Responses were collected via the standard Macintosh computer keyboard, with the 'Z' key used to indicate leftward motion and the 'M' key used to indicate rightward motion. The experiment was performed on a Macintosh computer using the Psychophysics Toolbox extension [14].

Procedure

The participants were instructed to fixate upon the red dot and then determine the direction of the moving dots. They were also instructed to complete the session as quickly and as accurately as possible. Each participant completed 1 session of approximately 60 minutes duration.

Each session consisted of 9 blocks of 200 trials, in each of which $P_{\rm A}$ remained fixed at 10%, 50%, or 90% (3 blocks for each condition). The order of the blocks was constrained to follow a Latin Square design. Participants were allowed a short break between blocks. To minimize anticipatory responding, response to stimulus intervals were drawn from a gamma distribution with a mean of 1 second for each trial, following the convention set in previous sequential RT tasks [89, 109, 19, 6]. Outlier RTs (less than 100 ms or greater than 900 ms, comprising less than 1.5% of the data) were not included in the analysis. We note that only the outlier was removed from the RT and error analysis; it was included in sorting RR, AR, RA, and AA sequences, since it precedes a trial that is included in the analysis. One participant failed to follow instructions and the corresponding data were omitted from the analysis.

During each block in the session, the subjects received the following feedback. Correct responses were denoted with a short beep sound, and error and premature, anticipatory responses were denoted with a buzz sound. In addition, on every fifth trial, the number of correct responses provided in that block so far was briefly flashed across the screen. This was the only feedback that was provided. Participants were seated at a viewing distance of approximately 60 cm from the screen. Our protocol in this experiment is similar to others in the literature (e.g., [82, 17, 95]).

3.1.2 Comparing model fits

The adapted DDM, described above in Section 2.1.2, was fitted to the RT and accuracy data of this experiment using the methods of Section 2.1.3. The resulting parameter values are displayed in the second row of Table 3.1, along with those obtained in fitting Experiment 1, which are displayed in the top row.

A study of the differences between the two tasks can lend some insight into the different fit parameterizations for each of the experiments. We note that the choice tasks presented in Experiment 2 are more challenging than those of Experiment 1 of [28], in which stimuli were highly discernable. The difference in signal to noise ratios ($\tilde{\mu}$) in the fits to the two experiments is therefore to be expected. In addition, more difficult tasks generally incur more conservative or cautious behavior in subjects, even when it is not in the subjects' best interests [10]. Increased caution (and consequently higher thresholds in DDM fits) have been shown to correspond to more difficult tasks (e.g., [98, 100, 99]). Thus, after correct responses in Experiment 2, model threshold adjustments (\tilde{z}_{down}) are small, whereas in the Experiment 1 they are larger, and corrections after errors (\tilde{z}_{up}) are smaller in Experiment 1 than in Experiment 2. Our Δ values are consistent with studies showing stimulus history dependence of up to 4 trials back (e.g., [114]). The remaining parameters are relatively closer in magnitudes for both experiments.

	$ ilde{\mu}$	$T_{\rm nd}$	k	$\tilde{x}_{0,\text{offset}}$	Δ	$\tilde{z}_{ m down}$	$ ilde{z}_{ m up}$	\tilde{z}_{\max}
Experiment 1	38.1747	0.2626	0.0943	0.0051	0.6860	0.0058	0.0348	0.2857
Experiment 2	19.3312	0.3359	0.1181	0.0034	0.6882	0.0034	0.1635	0.2062

Table 3.1: Comparison of DDM Parameterizations

3.2 Results

In order to better understand the relationship between sequential effects and error effects, data from Experiment 2 was sorted by stimulus sequence and response correctness and compared with model predictions as described in the previous chapter. We then analyze this data and ask how error and sequential effects are influenced by the relative frequencies of repetition (R) and alternation (A) trials. At the same time, we validate our model fits by comparing them with the data from Experiment 2.

In our discussion, we again refer to RA and AR sequences as *unexpected sequences*, and RR and AA sequences as *expected sequences*. The RT for an RA sequence is the RT corresponding to the A trial, and for an AR sequence, the RT corresponding to the R trial. As in Chapter 2, we call an *R line* one which connects plotted data for RR and AR, and an *A line* one which connects plotted data for RR and AR, and an *A line* one which connects plotted data for RA and AA. We consider only the two most recent trials in each sequence in our calculations, because the effects of errors are known to have a limited duration [89]. We note also that for the strongly biased stimuli ($P_A = 10\%$, 90%), longer sequences of A's, respectively R's, occur too rarely to yield sufficient data.

We now consider the role that alternation frequency plays in sequential and error effects. We first address overall trends in RT and ER, as shown in Figures 3.2(a) and 3.2(b), respectively, following the convention in the sequential effects literature (e.g. [113, 64, 28]). Trends for the $P_{\rm A} = 50\%$ blocks match trends from the unbiased random stimulus experiment with longer RTs and higher ERs for unexpected trials, and shorter RTs and lower ERs for expected trials, as we saw previously in Figure 2.1. Trends for the $P_{\rm A} = 10\%$ blocks and $P_{\rm A} = 90\%$ blocks are clearly distinguishable from those of the $P_{\rm A} = 50\%$ blocks, notably in the magnitudes of the slopes of R and A lines. Further, there is an approximate symmetry between the $P_{\rm A} = 10\%$ case and the $P_{\rm A} = 90\%$ case.

Figures 3.2(a,b) also reveal that sequential effects in mean RTs are strongly influenced by the probability of alternations, with respect to both overall mean RTs and ERs. Mean RTs for unexpected sequences (AR, RA) remain similar for all $P_{\rm A}$ conditions but there are significant differences in mean RTs for expected sequences (RR, AA). For the highest P_A , RT is faster on AA trials than the corresponding sequence RTs for lower P_{AS} , and for the lowest P_{A} , the RT is faster on RR trials than the corresponding sequence RTs for higher $P_{\rm A}$ s. As expected, we find that the effects of sequence $(F(3, 42) = 50.62, p < 0.001, \eta^2 = 0.26)$ and its interaction with $P_{\rm A}$ (F(3.36, 47.04) = 43.09, p < 0.001, $\eta^2 = 0.26$) on RT are both significant. Error rates are greatest for AR trials at the highest $P_{\rm A}$ and RA trials at the lowest $P_{\rm A}$. The effects of sequence $(F(1.95, 27.30) = 20.86, p < 0.001, \eta^2 = 0.31)$ and its interaction with $P_{\rm A}~(F(2.46, 34.44) = 20.19, \, p < 0.001 \, \eta^2 = 0.32)$ on ER are also both significant. The adapted DDM reproduces the qualitative patterns in the data, but overestimates RTs for expected sequences when their probabilities are low (RR, with $P_A = 90\%$; AA, with $P_A = 10\%$), and underestimates ERs for unexpected sequences (AR, RA): Figures 3.2(c,d).



Figure 3.2: Mean (a) RTs and (b) ERs for the three values of P_A in the biased random stimulus experiment, averaged over correct and error trials. The influence of P_A is most apparent in the mean RT plot on expected trials (RR, AA) and in the mean ER on unexpected trials (AR, RA). Model fits for (c) RTs and (d) ERs re-create behavioral trends in RTs and ERs but overestimate RTs for expected trials (RR, AA). The error bars in plots (a) and (b) represent the standard error of the mean, and in (c) and (d) the average value of standard error of the mean over 10 simulation runs (see Chapter 2, especially Section 2.1.3, for details).

We also found that the overall sequential effects are influenced by the probability of alternations. The relationship between the time to respond to sequences ending in R versus A on the final sequence is known to indicate relative preference for R or A trials [4]. Prior work has shown that preference for A trials varies with RSI, but the role of the likelihood of A trials in determining the relative preference for A has not been studied.

The green lines corresponding to $P_{\rm A} = 50\%$ in Figures 3.2(a,c) show no preference for R or A: expected sequences (RR, AA) yield faster RTs symmetrically in R and A than unexpected sequences (AR, RA). The red $P_{\rm A} = 10\%$ blocks show a strong preference for R: the mean RT after an R is faster in the case of RR than it is for AR, whereas the RT after A is similar for both RA and AA. The blue lines corresponding to $P_{\rm A} = 90\%$ show a strong preference for A: the RT after an A is faster in the case of AA than it is for RA, whereas the RT after an R is similar for both RR and AR. For $P_{\rm A} = 10\%$, the model predicts, as in the data, that the repetition RT is faster for RR than it is for AR, but the model predicts a slower alternation RT for AA than for RA, and it shows a symmetric trend for $P_{\rm A} = 90\%$. In summary, both data and model exhibit increases in preferences for R or A trials can be influenced by transition probabilities in addition to task properties such as RSI.

In Figures 3.3(a,b,c) we replot the mean RT data, separated into correct and incorrect responses, thus revealing differing sequential effects for each P_A . A two-way within-groups ANOVA shows that the effects of correctness (F(1, 14) = 249.64, p < 0.001, $\eta^2 = 0.80$), whether or not the trial was expected (F(1, 14) = 54.70, p < 0.001, $\eta^2 = 0.44$), and the interaction of these two factors (F(1, 14) = 88.38, p < 0.001, $\eta^2 = 0.62$) are all significant. For unbiased sequences ($P_A = 50\%$), sequential effects are again similar to those for correct and error trials in the unbiased random stimulus experiment (Figure 3.3(b), cf. Figure 2.2(a)). For both low and high P_A blocks, the orientations of the R and A lines are maintained,



Figure 3.3: (a-c) RT data for error and correct trials in Experiment 2 compared with the adapted DDM (d-f). The slopes of the R and A lines are reversed for correct and incorrect responses. Error trials incur faster responses on unexpected trials (RA,AR) than on expected trials (RR,AA); this trend is reversed for correct responses. For low $P_{\rm A}$, the R lines overlap, and for high $P_{\rm A}$, the A lines overlap, resulting in an approximate reflectional symmetry between data for the high and low $P_{\rm A}$ blocks, so that in some cases the mean time for an error trial is slower than or a correct trial. The error bars in plots (a-c) represent the standard error of the mean, and in (d-f) the average value of standard error of the mean over 10 simulation runs (see Section 2.1.3 for details).

with correct R lines sloping upwards from RR to AR and correct A lines sloping down from RA to AA. For $P_A = 50\%$, the slopes of the R and A lines for incorrect responses are nearly opposite the slopes of the R and A lines for correct responses.

In contrast, for $P_{\rm A} = 10\%$ blocks, the R lines cross and the A lines are further apart than in the $P_{\rm A} = 50\%$ blocks. For $P_{\rm A} = 90\%$ blocks, we see a mirrored trend, in which the A lines cross and the R lines are further apart than in the $P_{\rm A}$ = 50% block. We also note a striking asymmetry for the biased stimuli: for $P_{\rm A}$ = 10% R lines are, on average, closer together than A lines, and for $P_{\rm A} = 90\%$ this relationship is mirrored, so that A lines are closer than R lines. However, the mirroring is not perfect: the degree of overlap in R lines is greater for $P_{\rm A} = 10\%$ than the corresponding overlap in A lines for $P_{\rm A} = 90\%$.

The trends in correct and error trial RTs, including the crossover of the R and A lines, are generally captured by the adapted DDM, as shown in Figures 3.3(d,e,f). However, the steepness of slope of the R (respectively A) lines are underestimated for correct trials for $P_{\rm A} = 90\%$ (10%), due to overestimation of the RR (AA) RTs.

Next, we note that the sequential RT tradeoff between correct and error responses is also observed in Experiment 2, as shown in Figure 3.4(a). As in Figure 2.3, the areas of the circles are proportional to the corresponding ERs. The relationship between RTs for correct and error trials for each of the sequences RR, AR, RA, and AA is monotone (and nearly linear) for all points shown in the figure ($R^2 = 0.75$, p < 0.001), and this correlation is also captured by our model ($R^2 = 0.74$, p < 0.001): Figure 3.4(b). The sequences with the largest ER have relatively fast RTs for errors and relatively slow RTs for correct trials. Note, however, that data for individual $P_{\rm AS}$ of 10%, 50%, and 90% is not as strongly correlated. Differences in order can be expected because the sequential effects for each probability of alternation are influenced by the probability of alternation.



Figure 3.4: Sequential RT tradeoff for Experiment 2: mean RTs for correct trials are strongly correlated with mean RTs for error trials for each of the sequences RR, AR, RA, AA, for each value of P_A for both (a) data and (b) adapted DDM. The largest ERs are approximately 25%, and the smallest are approximately 1%.

Finally, we note that post-error slowing occurs for all P_A blocks with the same trend: the error trial itself incurs a slightly faster RT than the trial which precedes it, and the post-error trial incurs an RT significantly slower than RTs for the preceding two trials, as shown in Figure 3.5(a). A two-way within-groups ANOVA indicates that the effects of time before, upon, and after an error commission $(F(1.36, 19.04) = 68.25, p < 0.001, \eta^2 = 0.57)$ and on P_A $(F(2, 28) = 5.83, p < 0.01, \eta^2 = 0.07)$ are both significant, but the effect of their interaction is not significant. Thus, in Experiment 2, pre- and post-error RTs share the pattern of RTs in Experiment 1, and this pattern is preserved over all three values of P_A . Figure 3.5(b) shows that our model both qualitatively and quantitatively captures the post-error slowing in the Experiment 2. However, as in Experiment 1 (Figure 2.4), the model fails to produce the observed speed-up on the error trial itself.



Figure 3.5: (a) Post-error slowing in data for the Experiment 2 is independent of $P_{\rm A}$. (b) The model fit also predicts post-error slowing but does not fully account for pre-error speeding. The error bars in plot (a) represent the standard error of the mean, and in (b) the average value of standard error of the mean over 10 simulation runs (see Chapter 2 for details).

3.3 Discussion

Our adapted DDM predicts the characteristic trends in mean RTs for sequences of both unbiased and biased random stimuli. We show experimentally and for the first time that unexpected trials (AR or RA) result in relatively slow correct responses and fast errors, whereas expected trials (RR or AA) result in relatively fast correct responses and slow errors, as shown in Figure 3.3 (c.f. 2.2). Our model captures aspects of this behavior with the incorporation of post-error adjustments to the model thresholds: priming accounts for the sequential patterns in RT for correct trials, and error-correction accounts for the patterns in RT for the error trials.

The relationship between RTs for correct and error trials is central to our model: biasing the initial conditions towards expected sequences automatically biases them *against* unexpected sequences. Subjects biased against an unexpected stimulus will then respond to it slowly if they are to respond correctly, and rapidly if they are to respond in error. In contrast, in previous work [28, 66, 46], the biasing was instead applied to sensitivity to stimulus, so that the relationship between RT for error and correct trials was less direct. Moreover, when biasing is coupled with explicit post-error adjustments, further nuances in the relationship between mean time to respond correctly versus in error may be realized.

Significantly, we also identify a sequential RT tradeoff, in which the correlation between the mean RTs for error and correct trials for each of the sequences (RR, AR, RA, AA) is quite strong: a faster RT on an error response corresponds to a slower RT on a correct response. The correlation between mean RTs for correct and error trials is captured by our model, as shown in Figures 2.3 and 3.4.

The second experiment also shows that sequential effects in mean RTs overall, as well as in mean RTs for correct and error trials, are significantly influenced by the probability of alternations. Our data reveals remarkable near-mirror-symmetry between RT patterns for alternations when the probability of alternations is low and repetitions when the probability of alternations is high: incorrect responses are fast and correct responses are significantly slower. Sequential effects in ER also vary with the probability of alternations. Figures 3.2 and 3.3 show that our model captures this near-symmetry.

Moreover, we have shown, both in our data and in our model, that an increase in the likelihood of alternations corresponds to an increase in relative preference for alternations. This can be inferred from the RT versus sequence plots in Figure 3.2(a). The change in alternation preference with changing likelihood of alternations suggests that choice behavior can be informed and even manipulated by the probabilistic structure of the environment.

The sequential effects in RT and ER for various probabilities of alternation are of particular interest due to their relevance to prior physiological and imaging studies. In particular, previous work has shown that the anterior cingulate cortex (ACC) is sensitive to alternations in a sequence of stimuli and identified corresponding neural signals (e.g., [12]). Prior models of sequential effects, such as those of Jones et al. [66] and Gao et al. [46] have included a "conflict" signal modulated by activity in the ACC, and increasing in strength with frequent alternations. However, the near symmetry of behavior at high and low probabilities of alternations in our data suggests a comparable sensitivity to repetitions and alternations, rather than to alternations alone. Indeed, prior work has suggested that the role of the conflict signal in trials with long RSI, such as those considered in this study, is a minor one [66, 62], secondary to that of explicit error correction. Jones et al. [66] found that the incorporation of a conflict signal in their model resulted in a small but significant improvement in model fit. For short RSI, however, the role of response conflict is more significant [63, 61]. Future work could further clarify the respective roles of response caution (thresholds) and response conflict (ACC) co-varying RSIs and probabilities of alternation.

Additional directions for future work include a consideration of alternative error-correction and priming mechanisms. For example, the magnitude of adjustments made by our priming mechanism varies from trial to trial, while adjustments from the error-correction mechanism are consistent. Alternate models in which different update schemes are employed are worthy of consideration. Such a study could allow for further model simplification and provide a stronger account of behavior in choice tasks. Moreover, sufficient data should be gathered so that sequential and error effects can be studied and described for individual participants, by fitting RT distributions for different stimulus sequences and individual participants. Finally, a consideration of human behavior in more difficult tasks, such as those with low or variable stimulus discriminability, or tasks in which the probability of alternations varies during blocks of trials, can build upon our work.

In this study, we have presented a neurally plausible and conceptually straightforward account of sequential effects and post-error slowing by developing a simple repetition-based priming mechanism, coupled with an error-correction mechanism. We implemented these mechanisms within the context of a pure DDM, so the behavior can be described analytically and in closed form. Despite its simplicity, our implementation of the DDM accounts for nuances in behavior which are not found in previous models. In particular, we identified in our data, and our model accounted for, sequential effects for correct and error trials, as well as for trials during blocks with high and low probabilities of alternations. This suggests that an error-correction process, such as a simple adjustment of response thresholds after each trial, plays an instrumental role in sequential patterns in RT. Future work may further clarify the implementation of the error correction process and its implications for perceptual decision making tasks.

Chapter 4

Responses to Stimuli in Reward Maximization Tasks

4.1 Introduction

In this thesis we have so far considered fits of variants of the pure drift diffusion model (DDM) [10] and the Leaky Competing Accumulator Model [126] to human behavior in two alternative forced choice (TAFC) tasks in detail. We also briefly discussed properties of two additional models of TAFC behavior, the extended DDM and the Linear Ballistic Accumulator (LBA) models. We now note that although recent work has compared the predictions of the extended DDM and the LBA, no important qualitative differences between the abilities of the two models fit to behavioral data have been identified [39]. In this chapter we will compare the predictions of the pure and extended DDMs and the LBA in a task in which participants must trade off speed and accuracy in order to maximize their reward rates, and in doing so, we will identify significant differences between the two DDMs and the LBA.

Performance in reward maximization (Rmax) tasks is of interest and utility regarding the prediction and test of models. Prior to this work, however, Rmax task performance and corresponding model fits have been tested within models such as the DDM, but they have not yet been used to compare performance and fits across models. For example, the DDMs have been shown to model a tradeoff between speed and accuracy which is quite close to that of high performing participants [10, 110, 11, 6].

In this chapter, we compare the behavior of participants in an Rmax task as described by the two established DDM approaches with the description provided by the newer LBA model. We consider Rmax tasks in which participants complete TAFC blocks, and task difficulty is held constant within a block but varied between blocks.

The Rmax tasks have a common constraint: participants have only a fixed interval of time to attempt a block of TAFC trials, over which they may attempt to complete the task as many times as they are able. An experimental session will consist of several such blocks. Participants are instructed to adopt a strategy which yields the greatest possible rewards. In these tasks, payment for a participant in a given session is a function of her performance and of the number of trials completed. Correct responses are recognized with a set reward quantity, and incorrect responses may incur a punishment. The participant then decides with how much caution she will proceed. She may choose to attempt the task few times but do so very slowly and cautiously, or she may work faster but more carelessly. The best number of attempts for her will then be a function of a tradeoff between
speed and accuracy: when a participant increases her response speed, her accuracy will generally decrease, and vice versa. For very difficult tasks, the best tradeoff for the participant is clear: in the case of equally probable stimuli (the $P_{\rm A} = 50\%$ case of Chapter 3 and the only case considered in this chapter) she should guess randomly.

In this chapter, we consider three models, which include measures of evidence in favor of two options, or in favor of differences of evidence in favor of one option versus another. Again the overall preference for one option in a model is believed to be correlated with neural activity [50, 51, 49]. There is also a measure of the mean rate at which the state variable changes in value, represented by a drift rate, and a threshold level of certainty at which a decision is indicated, which we refer to as the caution parameter [39].

The caution parameter is critical to current understanding of Rmax task behavior. With high values of the caution parameter, speed is very low but accuracy is high. With low caution, the reverse is true. The caution is understood to vary with the difficulty of the task, and it is believed to be controlled by the participant. Caution in models is important; it can explain, for example, the relatively slow response times of elderly individuals [99]. This caution parameter is found to be a key factor in modulating the tradeoff between speed and accuracy in a task [10, 20, 6]. Caution levels can be deliberately manipulated within an experiment by presenting tasks of varied difficulty from block to block.

The pure and extended DDMs are known to account for Rmax behavior, both for participants who did and did not earn rewards at a near optimal rate via a deliberate manipulation of the caution parameter [110, 11, 6]. Diffusion models account for a difference in evidence noisily accumulated in favor of two choices: evidence in favor of one choice is presented as evidence against the other. The best level of caution can be determined analytically for the pure DDM and numerically for the extended model, as shown below.

Like the two DDMs, the LBA models the accumulation of evidence in favor of competing options. However, in the LBA, this process is considered separately for each of the alternatives, and the accumulation process, given an initial condition and drift, is deterministic on each trial, thereby motivating the terms "linear" and "ballistic". Drift rates and initial conditions are selected from distributions, but for thresholds there is only a single, constant value. The analytical simplification from a noisy process to a linear one has been shown to allow the LBA to capture much of the same behavior as the extended DDM, with fewer parameters [20, 39].

While the LBA has been applied to model behavior in tasks in which participants must choose between a fast response and a more accurate, slower response [20], the model has not been applied to study the tradeoff between speed and accuracy. Here we compare the LBA to the two DDMs as applied to models of Rmax task behavior. Prior work has claimed that the threshold of the LBA is analogous to that of the DDM and that the performance of the two models is comparable [39].

Direct numerical comparisons of the role of the thresholds in the models are straightforward. In each model, parameters can be fit to participant behavior at each difficulty level. For a given parameter set, changes in the speed and accuracy of response as caution is varied can be computed, and the optimal value of the caution parameter, the one which results in the highest reward rate, can then be computed at each difficulty level. Optimal behavior for the models and a speedaccuracy tradeoff for each of the participants can then be inferred, assuming that caution is deliberately varied from one difficulty condition to the next. In the case of the pure DDM, a unique parameter-free Optimal Performance Curve (OPC) describes the relationship between error rate (ER) and a normalized decision time (DT), yielding the greatest reward rate, independent of model parameters [10]. The pure DDM is known to be optimal in the sense that it delivers a decision of desired accuracy in the shortest possible decision time. Parameterized families of OPCs may also be determined for various parameterizations of the extended DDM, and as the values of its additional parameters (variances in drift rate, initial condition, and non-decision time, respectively) become small, the OPCs for the extended DDM approach that of the pure DDM.

Our results suggest that the DDM and the LBA models give very different accounts of behavior. While both models can reproduce key aspects of participant Rmax behaviors, the best DDM fits suggest that participants are on average least cautious on the most difficult tasks, in which the optimal strategy is random guessing. In contrast, the best LBA fit indicates that participants are on average more cautious on the most difficult task, and that they compensate by limiting the variance in initial conditions as the difficulty level decreases.

The structure of this chapter is as follows. In Section 2 we discuss our methodology: we describe the LBA and the pure and extended drift diffusion models as well as our approach to fitting each of the models. We also summarize existing optimal performance theory using the pure and extended DDMs and compare model performance in the limit of large noise. Section 3 describes our results, Section 4 contains a discussion of the results outlines conclusions and directions for future work. Additional fit details are relegated to an Appendix.

4.2 Methods

4.2.1 Comparing Processes: Drift Diffusion and Linear Ballistic Accumulation

In this section we describe in detail the pure and extended DDMs, as well as the Linear Ballistic Accumulator Model.

Pure Drift Diffusion Model

We first consider the pure DDM with unbiased initial conditions. The evolution of the difference in evidence for the two choices is governed by the following equation:

$$dx = \mu dt + \sigma dW, \quad x(0) = x_0. \tag{4.1}$$

Evidence accumulates noisily from $x(0) = x_0$ to the first time T at which x(T) = +z or -z. The two thresholds, +z and -z, correspond to thresholds for selecting the correct or incorrect choice on a given task, respectively. We note that the pure DDM in this case requires only 4 parameters to predict DT: a drift rate μ , an initial condition x_0 , a Weiner noise component with variance σ^2 , and a threshold or caution parameter z. In addition, the pure DDM is augmented by a nondecision time parameter, T_0 , corresponding to factors independent of the decision making process, and the estimated reaction time from the pure DDM is the sum of the decision and nondecision times, so $RT = DT + T_0$.

In this chapter, we continue to focus on the role of the caution parameter z. We note that the caution parameter can take on any nonnegative value. High caution corresponds to longer decision times but also an increase in accuracy. For the pure DDM, we have closed form analytical expressions for DT and ER [10]:

DT =
$$\frac{z}{\mu} \tanh\left(\frac{z\mu}{\sigma^2}\right) + \frac{2z}{\mu} \cdot \left(\frac{1 - \exp\left(\frac{2x_0\mu}{\sigma^2}\right)}{\exp\left(\frac{2z\mu}{\sigma^2}\right) - \exp\left(-\frac{2z\mu}{\sigma^2}\right)}\right) - \frac{x_0}{\mu},$$
 (4.2)

$$ER = \frac{1}{1 + \exp\left(\frac{2z\mu}{\sigma^2}\right)} - \left(\frac{1 - \exp\left(\frac{2x_0\mu}{\sigma^2}\right)}{\exp\left(\frac{2z\mu}{\sigma^2}\right) - \exp\left(-\frac{2z\mu}{\sigma^2}\right)}\right).$$
(4.3)

We note that in this chapter we again allow nonzero initial conditions in our model, as we did with the pure DDM in Chapters 2 and 3, in which initial conditions played a critical role in the trial to trial RT patterns in the system. However, in this chapter we are only interested in mean RTs and save for future work the question of trial to trial variability, or biased stimuli [10, 110]. We allow nonzero initial conditions in the DDM in order to provide a more direct comparison with parameters in the LBA. As we will see later in this chapter, the LBA model design ensures nonzero initial conditions for a majority of trials.

Extended Drift Diffusion Model

In the extended DDM, the evolution of an individual trial is governed by the same process as the pure DDM, but with added variability in initial condition, drift rate, and non-decision time, so that new values for these parameters are selected upon the start of each trial. The evolution of the extended DDM is governed by the following equation:

$$dx = \mu^* dt + \sigma dW, \quad x(0) = x_0^*, \tag{4.4}$$

in which accumulation of evidence proceeds until one of the thresholds $\pm z$ is reached. Here μ^* , σ^2 , z, x_0^* , and T_0 represent the drift rate, variance, threshold, initial condition, and non-decision time for a given trial, respectively. For each trial, μ^* is selected from $\mathcal{N}(\mu, s_{\mu}^2)$, x_0^* is selected from $\mathcal{U}(x_0 - \frac{s_{x_0}}{2}, x_0 + \frac{s_{x_0}}{2})$, and T_0^* is selected from $\mathcal{N}(T_0, s_{T_0})$, where \mathcal{N} and \mathcal{U} respectively denote Gaussian (normal) and uniform distributions.

This additional variability in parameter values allows for augmented descriptive power. In particular, the extended DDM, unlike the pure DDM, can predict different RT distributions for correct and error trials, even with unbiased mean initial conditions. Prior work has suggested that the parameters added in going from the pure to the extended DDM sufficiently extend the descriptive capabilities of the model to merit the additional modeling cost [96, 97, 6]. However, simple analytical, closed-form expressions for DT and ER do not exist for the extended DDM, as they do for the pure DDM.

We will focus again on the role of the caution parameter in the context of the extended DDM, as we have done in the context of the pure DDM. Caution can be manipulated by a participant to weight responses in favor of speed or accuracy. The threshold z can assume any nonnegative value outside the range of initial conditions [125].

Linear Ballistic Accumulator Model

The LBA model is conceptually quite simple and yet has been shown to provide rich descriptions of behavior, rivaling those of the extended DDM [20, 39]. In this model evidence for each of two or more choices $x_i(t)$ accumulates *linearly* and *ballistically* towards a threshold z at drift rate μ^* :

$$x_i(t) = x_{i0}^* + \mu_i^* t, \quad i = 1, 2, \dots, N.$$
 (4.5)

As in the extended DDM, there is variability in parameters from trial to trial. In the LBA model, μ_i^* is selected from $\mathcal{N}[\mu_i, s]$, x_{i0}^* is selected from range $\mathcal{U}[0, A]$ on each trial, in which A is the maximum value that an initial condition x_{i0}^* may assume from the distribution. The fastest accumulator $x_i(t)$ to reach the threshold z is selected. In prior work, A has been restricted to lie below z: A < z [20, 39]. We note that while the drift rates are generally different for each accumulator $(\mu_i \neq \mu_j)$, the remaining parameters (A, z, s, T_0) are common to each of the accumulators.

Closed form expressions have been derived to describe overall behavior in the LBA in [20]. First, the cumulative distribution function (CDF), $F_i(t)$, and probability density function (PDF), $f_i(t)$, can be written in terms of the LBA parameters for individual accumulators.

$$F_{i}(t) = 1 + \frac{z - A - t\mu_{i}}{A} \Phi\left(\frac{z - A - t\mu_{i}}{ts}\right) - \frac{z - t\mu_{i}}{A} \Phi\left(\frac{z - t\mu_{i}}{ts}\right) + \frac{ts}{A} \phi\left(\frac{z - A - t\mu_{i}}{ts}\right) - \frac{ts}{A} \phi\left(\frac{z - t\mu_{i}}{ts}\right), \qquad (4.6)$$

$$f_{i}(t) = \frac{1}{A} \left[-\mu_{i} \Phi\left(\frac{z - A - t\mu_{i}}{ts}\right) + s\phi\left(\frac{z - A - t\mu_{i}}{ts}\right) + \mu_{i} \Phi\left(\frac{z - t\mu_{i}}{ts}\right) - s\phi\left(\frac{z - t\mu_{i}}{ts}\right)\right], \qquad (4.7)$$

where $\Phi(\cdot)$ is the corresponding cumulative distribution function over a normal distribution and $\phi(\cdot)$ is its probability density function. Derivations of Eqs. (4.6-4.7) can be found in the Appendix of [20].

To determine the mean first passage time for these competing accumulations, we use the *defective PDF*, $PDF_i(t)$, so named because it generally integrates to a value between 0 and 1, unlike the standard PDF which integrates to 1. The defective PDF describes the likelihood that accumulator $x_i(t)$ reaches the threshold provided that one of the other accumulators has not already done so:

$$PDF_i(t) = f_i(t) \prod_{j \neq i} (1 - F_j(t)).$$
 (4.8)

However, because the drift rates μ_i are selected from a normal distribution, in some cases all drift rates selected will be negative, corresponding to an infinite deliberation period, and in this situation no response will be given.

In order to compare responses to those predicted by the two DDMs, which yield finite response times on every trial, we consider only simulated LBA trials which yield a finite response time. To do this, we modify the above LBA expressions by a normalization factor of $\alpha(\mu_1, \ldots, \mu_N, s) = 1 - \prod_{i=1}^N \Phi\left(-\frac{\mu_i}{s}\right)$ corresponding to the probability that none of the accumulators reach threshold, so that in a TAFC we have $\alpha(\mu_1, \mu_2, s) = 1 - \Phi\left(-\frac{\mu_1}{s}\right) \Phi\left(-\frac{\mu_2}{s}\right)$. (These expressions follow from the probability that a given accumulator has a negative drift rate: $\Phi\left(\left(-\frac{\mu_1}{s}\right)\right)$. The normalized defective probability density functions are given in [20] as

$$p_i(t) = \frac{\text{PDF}_i(t)}{\alpha(\mu_1, \dots, \mu_N, s)}.$$
(4.9)

For a two choice task, we have then have $p_1(t)$ and $p_2(t)$ defined as

$$p_1(t) = \frac{\text{PDF}_1(t)}{1 - \Phi\left(-\frac{\mu_1}{s}\right) \Phi\left(-\frac{\mu_2}{s}\right)},$$
 (4.10)

$$p_2(t) = \frac{\text{PDF}_2(t)}{1 - \Phi\left(-\frac{\mu_1}{s}\right)\Phi\left(-\frac{\mu_2}{s}\right)}, \qquad (4.11)$$

such that $\int_0^\infty (p_1(t) + p_2(t)) dt = 1$. We may then use the following expressions for DT and ER:

DT =
$$\int_0^\infty t(p_1(t) + p_2(t))dt,$$
 (4.12)

$$ER = \int_0^\infty p_2(t)dt. \qquad (4.13)$$

We will again focus upon the role of the threshold or caution parameter in this model. Unique to the LBA as described in the literature [20, 39] is the restriction that the threshold must not lie within the range of initial conditions, such that $z \ge A$. As a result, the LBA, unlike the two DDMs, almost always yields nonzero DTs. The implications of this restriction on the LBA in determining an optimal speed-accuracy tradeoff for the LBA is discussed in the following section.

4.2.2 Optimal Performance in the Models

We focus our study of diffusion as well as accumulator models upon the question of optimal performance in time-limited blocks which compose TAFC tasks in which participants receive a unit of reward for each correct response. We define optimal performance as a strategy which maximizes the Reward Rate (RR), defined as:

$$RR = \frac{1 - ER}{DT + T_0 + RSI}.$$
(4.14)

We wish to determine the relationships between ER and DT which yield the greatest possible RR for a given decision making model. The relationship between ER and DT at the optimal RR lies along an Optimal Performance Curve (OPC) [10], which relates normalized DT to ER. Here normalized DT is defined as $\frac{\text{DT}}{\text{D}_{\text{tot}}}$, where D_{tot} is defined as $D_{tot} = T_0 + RSI$. D_{tot} , the total time between successive responses, includes non-decision time T_0 and the response to stimulus interval (RSI).

The OPC is a relationship describing the tradeoff between speed (the normalized DT) and accuracy (1 - ER) which must hold in order to yield the maximum RR for a task in a specified condition.

An OPC, or a family of OPCs can be determined for various TAFC models. Intuition can help explain the general shape of the curve. In the limit of very noisy stimuli, prolonged evidence accumulation cannot improve much over random decisions and the best threshold to choose will be the lowest available, which results in the smallest DT, with ER \approx 0.50. Alternatively, a very easy task with a very salient stimulus requires only a small amount of accumulation to achieve a high degree of accuracy, so that DTs are also small, but ER \approx 0. This low-difficulty condition results in the highest RRs. Intermediate difficulty stimuli require more evidence accumulation, and hence higher thresholds, yielding longer DTs and ERs intermediate between 0 and 0.50. In all cases and conditions, the selection of a non-optimal threshold will result in suboptimal performance, so that the maximum available RR is not realized. Examples of OPCs appear in Figures 4.1 and 4.2, below.

Optimal Performance under the Pure DDM is Described by a Unique Curve

The pure DDM is unique in that it has a parameter-free OPC, defined by the function:

$$\frac{\mathrm{DT}}{\mathrm{D}_{\mathrm{tot}}} = \left(\frac{1}{\mathrm{ER}\log\frac{1-\mathrm{ER}}{\mathrm{ER}}} + \frac{1}{1-2\mathrm{ER}}\right)^{-1},\tag{4.15}$$

as derived in [10]. This function, which relates two key behavioral quantities is illustrated in solid black in Figures 4.1 and 4.2(a,b). See [10, 134] for derivations and further discussion of the OPC.

Optimal Performance under the Extended DDM is Not Uniquely Defined

The extended DDM has families of OPCs rather than a unique OPC, as in the pure DDM. In the extended DDM, variability in initial conditions precludes the possibility of trials with a DT = 0. However, for low values of the variance parameters in the extended DDM, the OPCs for the extended DDM approach the OPC for the pure DDM. A sample OPC for the extended DDM is illustrated in the dashed line in Figure 4.1. To compute these curves, we fixed all parameters except drift rate and threshold. For each drift rate, we found the threshold which optimized RR and used this threshold to determine ER and normalized DT. For more details, the reader is referred to [10].



Figure 4.1: Simulated Optimal Performance Curves (OPCs) for the pure DDM (solid black) and extended DDM with varied initial conditions (dashed black). Variability in initial conditions in the extended DDM results in nonzero thresholds and hence DTs for easy stimuli when the ER \rightarrow 0, unlike in the pure DDM or the LBA. In the standard LBA, variability in initial conditions requires nonzero thresholds even for difficult tasks, such that ER \rightarrow 0.5, see Figure 4.2. The above figure was generated using a variability in initial conditions, s_x , of 0.14 and was adapted from [10, Figure 14].

Optimal Performance Curve under the LBA is Not Uniquely Defined

Analogous families of OPCs can be defined for the LBA, as shown in Figure 4.2. The LBA expressions of Eqs. (4.6-4.11) are complicated, and simple analytical expressions of the corresponding OPC families are not available. However, approximation of the family of OPCs is possible. To do this, we fix T_0 , RSI, s, $\mu_1 + \mu_2 = 1$ and choose A. We consider a range of $\mu_1 > \frac{1}{2}$; for each μ_1 we can then calculate ER and DT, and from them, we can estimate the optimal z and find the corresponding ER and DT, producing a point on the OPC for the given A value.

We find that a different OPC is generated for each value of A, showing that there is no unique OPC for the LBA model.

This observation is consistent with the observation that, for any $\mu_1 = \mu_2 = \mu$ (equal evidence for both options), the expected accuracy will be exactly 0.50 and no additional accuracy may be realized or information accumulated over time. It follows that the optimal threshold is then the lowest possible threshold. For the pure DDM and the extended DDM with zero initial condition variance ($s_x = 0$), the threshold parameter, z, can go to 0, while in the LBA, the threshold must lie at or above the top of the range of initial conditions, i.e., $z \ge A$. The lowest threshold, and the optimal threshold for $\mu_1 = \mu_2 = \mu$ is then z = A. The OPC curve plotting $\frac{DT}{DT_{tot}}$ therefore varies with the value of A as shown in Figure 4.2(a): the smooth portions of each curve correspond to z > A (on the left) and z = A(on the right). We see that there are multiple OPCs for the LBA, one for each value of A; moreover, unlike the OPC for the DDM, these curves terminate at ER = 0.5 with finite normalized DTs.

We therefore also consider an *adapted version of the LBA*, such that the thresholds are allowed to fall within the range of initial conditions as well as above it, and show that the OPC for this adapted LBA is also not unique. If one of the initial conditions in a trial falls above the threshold value, the DT for that particular trial is then 0. If both of the initial conditions in a trial are above the threshold, then the option corresponding to the larger initial condition is selected, and the decision time is again 0. The mean DT and ER, ER_a and DT_a, for this adapted



Figure 4.2: Simulated Optimal Performance Curves (OPCs) for the LBA. We see that OPCs for both the standard LBA (a) and adapted LBA, in which threshold z is allowed to go to 0 (b) are not unique. The following parameters were used in the simulation: s = 0.32, $T_0 = 226$ ms, RSI=1000ms, $\mu_1 + \mu_2 = 1$. The OPC predicted by the pure DDM is shown in black.

system with z < A are defined accordingly:

$$\operatorname{ER}_{\mathrm{a}}(A, z, \mu_{1}, \mu_{2}, s) = \frac{1}{2} \cdot \frac{A - z}{A} + \frac{z}{A} \cdot \operatorname{ER}(z, z, \mu_{1}, \mu_{2}, s), \qquad (4.16)$$

$$DT_{a}(A, z, \mu_{1}, \mu_{2}, s) = \frac{z}{A} \cdot DT(z, z, \mu_{1}, \mu_{2}, s), \qquad (4.17)$$

in which ER and DT are calculated as above for the standard LBA model.

Numerically derived OPCs for this adapted LBA are shown in Figure 4.2(b). In the case of $\mu_1 = \mu_2 = \frac{1}{2}$, it allows the optimal threshold $z^* = 0$, and so limits in the desired point at which DT is 0 and the ER is 0.50. This is because a higher value of z would not result in any change in accuracy, so the best value of z is the one which minimizes DT. Moreover, as $\mu_1 \rightarrow \mu_2$, we expect $z^* \rightarrow 0$. Similarly, for $\mu_1 >> \mu_2$, we expect low ER and corresponding low DT, due to high bias towards correct choices. The smooth portion of the curve to the left starting at ER=0 corresponds to z > A, and that to the right ending at ER=0.5 corresponds to z < A. The kink between these segments represents a jump between endpoints of these two ranges for z. For this parameterization and simulation, z = A is never optimal. The adapted LBA predicts that optimal behavior will not result in ERs lying between these two ranges.

In the degenerate case A = 0, no additional accuracy is gained by having a nonzero z as there is no noise in initial conditions, so that z^* is always 0, not just at the case $\mu_1 = \mu_2 = \mu$. Thus, as $A \to 0$, we expect that $z^* \to 0$, even when $\mu_1 \neq \mu_2$: OPC curves get flatter for every ER, such that the normalized DTs get smaller and smaller, as seen in the plot.

Noise Scales Differently in the pure DDM and LBA

We compare noise scaling in the two models and note that as noise increases, the models behave differently. Noise levels can be high in certain TAFC settings, such as those considered later in this chapter. The signal to noise ratio is varied from one condition to the next, but this variation can be accounted for in the models by allowing mean drift rates to change, so that comparisons of the role of noise alone in the DDM and LBA should be meaningful. We consider additive noise in the pure DDM, and in the LBA we also consider variability in both the drift rate s and range of initial conditions A. As the noise parameters are intended to remain analogous between models, differences between noise in the models is noteworthy.

Scaling in the Pure DDM for High Noise

We consider approximations of $\text{ER}(\frac{1}{\sigma^2})$ and $\text{DT}(\frac{1}{\sigma^2})$ about the point $\text{ER}(\frac{1}{\sigma^2}=0)$ and $\text{DT}(\frac{1}{\sigma^2}=0)$ We can Taylor expand the DDM expressions with respect to small parameter $\frac{1}{\sigma^2}$:

ER =
$$\frac{1}{2} - \frac{z\mu}{2} \left(\frac{1}{\sigma^2}\right) + \frac{z^3\mu^3}{6} \left(\frac{1}{\sigma^6}\right) + O\left(\frac{z^5\mu^5}{\sigma^{10}}\right)$$
 (4.18)

DT =
$$z^2 \left(\frac{1}{\sigma^2}\right) - \frac{\mu^2 z^4}{3} \left(\frac{1}{\sigma^6}\right) + O\left(\frac{\mu^4 z^6}{\sigma^{10}}\right)$$
 (4.19)

We see that ER is O(1), and DT is $O\left(\frac{1}{\sigma^2}\right)$ with respect to $\frac{1}{\sigma^2}$: ER scales differently with high noise σ than does DT. However, both ER and DT are of the same order with respect to σ^2 . In particular, taking limits as $\sigma \to \infty$, we find that ER $\to 0.5$ and DT $\to 0$. Intuitively, we see that large noise pushes the accumulation process very rapidly away from the initial condition and towards one of the boundaries. For difficult tasks, we then expect ER=0.5 and DT=0 to lie on the OPC for the pure DDM, as it does.

We now consider the scaling of ER and DT with noise parameters in the LBA. We will see that large noise in the LBA generally leads to nonzero DTs.

Scaling in the Standard LBA in the case $\mu_1 = \mu_2$

The LBA has two sources of noise or variability in the case of non-discriminable stimuli $(\mu_1 = \mu_2 = \frac{1}{2})$. The first source of noise is the variability in drift rates, s, and the second is the variability in initial conditions, A.

In both cases, as the two drift rates μ_1 and μ_2 are equivalent, and as drift is the only source of bias in the LBA model, the ER is 0.5. We note that the mean DT

can be 0 if and only if A = 0. This restriction on the DT follows from the constraint that $z \ge A$, so that a nonzero A implies a nonzero mean distance to accumulate to threshold. To see this, first suppose that s = 0 and then allow s to increase, producing a distribution of drift rates centered around $\mu_i = \frac{1}{2}$. Moreover, as Aincreases, the minimum allowable value of the threshold z also increases because $z_{min} = A$. It follows that for $\mu_1 = \mu_2$ and the minimum z, DT increases with the value of A.

These behaviors for $\mu_1 = \mu_2$ and high A or s are quite different from those seen in the pure DDM, in which large noise implies $DT \rightarrow 0$.

4.2.3 Reward Maximization Experiment

We consider here human performance in a motion discrimination task previously presented and analyzed in [6]. Participants (n = 17, 6 male) were tasked to discriminate the direction of moving dots on a computer screen. The participants viewed the stimuli at approximately 2 feet from the CRT monitor on which they were presented and indicated the direction of motion via keypress on a standard keyboard. Participants could indicate leftward motion with the 'Z' key and rightward motion with the 'M' key. The experiments were conducted at a Macintosh computer, using the Psychophysics Toolbox [14]. Each participant completed at least 13 daily sessions of 60 minutes duration.

The first four sessions involved training and practice activities in which participants did not receive monetary reward. In later sessions, participants first completed five blocks of a motion discrimination task, with each block presented at a different coherence (0, 4, 8, 16, and 32%, randomized across participants); participants earned \$0.02 for each correct response. Performance improved markedly over the first 5 sessions and for certain participants continued to improve until session 9. Here we only use data from sessions 10-13. Premature responses, either anticipatory or with RTs of less than 100 ms, were penalized with a buzzing sound and a 4 second timeout period. When participants did not respond prematurely, a response to stimulus interval was selected from an exponential distribution with mean of 1 second.

After they completed the free response motion discrimination task, participants performed an interval timing task and then a signal detection task. The setup of the signal detection task was the same as that of the Rmax task, except participants were instructed to indicate the onset of dot motion, regardless of its direction. In one block they would be instructed to press the 'M' key, and in the other, the 'Z' key. They again received \$0.02 for each correct (non-anticipatory) response. For more details, the reader is referred to [6].

In this chapter, we consider only the results of the reward maximization and signal detection tasks.

4.2.4 Data Fitting Procedures

Fits were performed to participant data from the task above for the two DDMs and the LBA using published toolboxes for the models in Matlab [125] and R [38], respectively. Fitting the data to the LBA model required some modifications to the standard LBA code, as outlined in [38]. Data were separated for individual participants by difficulty condition. Fits were computed for each individual participant over all five difficulty conditions. Multiple fits were performed for each condition and participant, first varying only caution (threshold) with difficulty level, and then varying the range of initial conditions as well, while the remaining parameters were held constant. The same data and partitioning of data, was used for both model fit toolboxes.

The DDM and LBA models in this chapter were fit separately to distributions of RTs for correct and error trials in each condition because the two fit toolboxes have been designed to fit properties of distributions of RTs rather than mean values. A primary goal of the work in this chapter is to compare the efficacy of the models, and the least biased way to do this is via the corresponding toolboxes designed to fit the models. Both toolboxes allow the user to set constraints such that certain parameters are held constant while others are allowed to vary from one condition to the next. The DDM toolbox does this by using a combination of a system of matrix equations similar to those in general linear models coupled with post-processing to remove outliers [127, 128]. The LBA toolbox uses the quantile maximum probability estimator method [55] to estimate the parameters of density functions for distributions for correct and error trials, and these parameters can then be used to select model parameters. In all cases, fits were performed on the 10-90th quantiles for correct and error trials. In contrast, the models in Chapters 2 and 3 were fit only to mean RT and ER for correct and error trials data using the least squares method.

The goodness of fit for each model prediction of mean RT and ER for each condition and participant was then assessed using Akaike, Corrected Akaike, and Bayesian Information Criteria, as described in the first chapter of this thesis. Finally, in addition to modeling the way participants actually performed on the task, their theoretical optimal performance for each difficulty condition or drift rate could be estimated by allowing the caution parameter to vary freely while holding all the other parameters constant. The optimal value of the caution parameter is defined as that which results in the highest possible reward rate (a combination of rapid RT and/or low ER), given fixed (fitted) values for the remaining parameters.

4.3 Results

In order to determine relative suitability of DDM and LBA models for reward maximization task data, we analyze fits in several ways. We fit each individual participant in the following models:

- A pure DDM, in which only the thresholds z vary between the coherence conditions.
- An extended DDM, in which only the thresholds z vary between coherence conditions.
- A second extended DDM, in which both the thresholds z and range of initial conditions s_x vary between coherence conditions.
- A LBA model, in which only the thresholds z vary between the coherence conditions.
- A second LBA model, in which both the thresholds z and the range of initial conditions A vary between coherence conditions.

Fits to mean RT and ER data for each participant and condition were quite good for each model: the mean value of the correlation coefficient, R^2 , was greater than or equal to 0.95 for each of the models. Mean AIC, AIC_c, and BIC values for the fits to each participant, averaged over all participants, are illustrated in Figure 4.3 and summarized in Table 4.1, along with the correlation coefficient. The best overall fit to mean RT and ER data was found in the pure DDM.

		1.7.01	1.7.01		
Model	Total Parameters	AIC	$ AIC_c $	BIC	R^2
LBA	13	80.44	15.44	110.37	0.95
LBA with Var. ICs	17	66.18	23.68	105.32	0.99
Pure DDM	13	63.57	-1.43	93.50	0.98
Extended DDM	16	66.37	20.66	103.21	0.98
Extended DDM with Var. ICs	20	66.76	30.40	112.81	0.99

Table 4.1: DDM and LBA Model Fit Comparison

Metric values for each participant and model were computed based on mean RT and ER data (illustrated separated by participant and condition in the Appendix, in Figures 4.10 and 4.11, respectively). The metric scores for each model were then averaged over all participants and conditions to determine mean scores for each model. The metric values and model fits to RT and ER suggest good predictions of RR behavior. Fits to RR data by participant are shown in the Appendix in Figure 4.12.

Superior AIC/BIC scores for pure DDM fits may be attributed in to the fact that the pure DDM uses fewer parameters than all of the other models in this study except for the standard LBA. Fit metrics reward goodness of fit while penalizing the use of extra parameters. Smaller values of the metric scores are desirable and negative values are possible [1, 2, 3]. Many of the other models achieved slightly higher mean R^2 values than the pure DDM, but they did so with a greater number of parameters, and so biased model comparison scores in favor of the pure DDM. We note some differences between the fit scores: allowing the range of initial conditions to vary in the LBA results in a superior fit with regard to each of the metrics except AIC_c . However, for the extended DDM, allowing the range of initial conditions to vary does not improve model fits according to any of the metrics.

We note that we are fitting a small data set of mean RT and ER measurements for each subject spread over five difficulty conditions. We consider RT and ER because they are traditionally of greatest interest in modeling Rmax task behavior. When comparing the pure DDM fit to the two extended DDM fits over the entire distribution of RTs instead using the DDM toolbox, the extended DDM ($\chi^2 = 168.35$, p < 0.05) and extended DDM with variable initial conditions ($\chi^2 = 408.94$, p < 0.001) yield superior deviance scores [27]. However, fitting the entire distribution is not a goal of this work, so for our purposes the pure DDM provides the superior diffusion model fit.

Fits for each of the models and participants provided good qualitative matches to the ER and corresponding estimated normalized DT data, for which nondecision times T_0 were estimated for each participant from the mean RTs for the fastest 25% of their signal detection trials, as in [6, 5].

The high performing participants generally had ERs and normalized DTs close to those on the OPC for the pure DDM. Figure 4.4 shows fits to the behavior for one representative higher performing and one lower performing participant (Subjects 32 and 34). Figure 4.5 shows the means averaged over all subjects. In each figure, the data is shown in solid black with error bars, and fits are superimposed in curves of various colors. As with individual subjects, the average behavior trends away from the OPC for the pure DDM as ERs increase. The LBA models and the pure DDM overestimate normalized DTs and the extended DDMs slightly



Figure 4.3: Mean of AIC, AIC_c , and BIC scores for all participants in each of the model fits. Better fits have lower scores. Error bars indicate standard error of the mean values, calculated from the fits to the participant RT and ER data shown in the Appendix in Figures 4.10 and 4.11.

underestimate them. This is due in part to some subjectivity in the estimation of nondecision time: the LBA tends to fit smaller values than do the DDMs [39]. However, while the LBA fits lie above the data curve, the optimal LBA fit lies below it. The optimal LBA fits were found by numerically finding the best threshold for each drift rate. For high performers, such as Subject 32, the range of initial conditions is small, so that thresholds can be small without a major sacrifice in accuracy. The difference between this empirical normalized DT and the corresponding normalized model DT (the OPC for the pure DDM) was a good predictor of overall RR for each of the difficulty conditions ($R^2 = 0.53$, p < 0.001). Fits for all participants can be found in the Appendix in Figure 4.13. In each of the panels of the figure, we also included an LBA OPC, which represents optimal behavior given the participant's estimated parameters for drift, variability in drift, range of initial conditions, and nondecision time, from which the optimal threshold can be numerically determined. We note that for many high performing participants, including Subject 32, the predicted LBA OPC lies below the model fits to behaviors for intermediate ERs. We will see later that this is to be expected due to restrictions placed on the range of initial conditions A to match both long and short RTs in different difficulty conditions.

In order to better understand differences between the model fits, we considered the mean parameter fits for the caution parameter. That is, for each participant, difficulty condition, and DDM fit, we calculated a threshold, and then we averaged over all the individual threshold values for a given model and difficulty condition. The resulting mean threshold values are shown in Figure 4.6. We see that on average, for the pure DDM and extended DDM with fixed range of initial conditions for all coherences, the threshold values increase with coherence. Allowing the range of initial conditions in the DDM to vary with coherence also produces thresholds that vary significantly with coherence (F(4, 64) = 73.72, p < 0.01, $\eta^2 = 0.82$).

We next compare mean values of the threshold parameter, averaged across participants, in the two LBA model fits in Figure 4.7. In the first LBA fit, only the threshold is allowed to vary by condition, and the trend in participant caution is consistent with that shown by the first two DDMs: thresholds modestly increase



Figure 4.4: Comparative data and fits to the OPC for (a) high performing and (b) low performing participants. The OPC for the LBA for a given participant as described in Section 4.2.2 is also computed and shown, in purple open circles and for the pure DDM in grey. Empirical normalized DTs, estimated from mean RT data for the main task as well as RTs from a signal detection task, are shown in black with standard error bars. The LBA fits overestimate normalized DTs for both subjects. The LBA OPC, however, lies below the data, most notably for Subject 32, because the fits for Subject 32 require a small range of initial conditions to get low DTs in difficult tasks, and the initial condition variance A is fixed, leading to small normalized DTs throughout.

with coherence level. However, for the second LBA fit, in which the range of initial conditions are allowed to vary by condition, this trend is reversed: thresholds decrease with coherence level. This difference is significant: the results of a withingroups ANOVA on parameter values for the LBA with and without variability in the A values show that the main effect of the model type $(F(1, 16) = 5.62, p < 0.05, \eta^2 = 0.01)$ and the interaction of model type and difficulty condition $(F(4, 64) = 4.29, p < 0.01, \eta^2 = 0.08)$ are both significant. In other words, the LBA fit behavior is determined by both difficulty condition and model type. Values



Figure 4.5: Data and fits to Rmax behavior averaged over all participants in Figure 4.13 is shown, along with a comparison to OPCs for the pure DDM and the LBA models. Data is shown in the thick black line, with mean values representing the mean of the mean DTs for each of the participants, and error bars the mean of the standard errors for each of the participants for a given condition. The LBA fit curves overestimate mean normalized DTs, but the optimal LBA curve underestimates them.

of the thresholds for each individual participant and coherence level for each of the models are shown in the Appendix in Figure 4.14.



Figure 4.6: Mean threshold parameter values for the three DDM fits, averaged by difficulty condition. The pure (blue) and first extended DDM (red) fits do not differ significantly; only the effect of difficulty condition is significant (F(4, 64) = 3.91, p < 0.01, $\eta^2 = 0.16$). Bars indicate standard errors for n = 17 subjects. Varying the range of initial conditions in the second, extended DDM fit (green) does not result in better fits by AIC/BIC. However, when this model fit is compared to the other two DDM fits, the main effects of model (F(2, 32) = 3.99, p < 0.05, $\eta^2 = 0.02$) is significant, as well as the interaction of model type and difficulty condition (F(8, 128) = 2.06, p < 0.01, $\eta^2 = 0.01$).



Figure 4.7: Mean threshold parameter values differ significantly for the two LBA Model fits. Bars indicate standard errors for n = 17 subjects. In the first fit (blue), only drift (μ_1) and threshold (z) parameters are allowed to vary by difficulty condition. In the second fit (red), the range of initial conditions (A) is allowed to vary between difficulty conditions: this results in significantly better model fits (see text). However, these two fitting procedures tell different stories: as difficulty increases participants become less cautious in the first model and more so in the second. In particular, the results of a within-groups ANOVA on parameter values for the LBA with and without variability in the A values shows that the main effect of the model type (F(1, 16) = 5.62, p < 0.05, $\eta^2 = 0.01$) and the interaction of model type and difficulty condition (F(4, 64) = 4.29, p < 0.01, $\eta^2 = 0.08$) are both significant.

We next consider the role of variability in initial conditions. Mean values of the range of initial conditions, averaged across all participants for both the DDM and LBA models are shown in Figure 4.8. The two models in which variability in initial conditions are allowed show a similar trend: mean variability in initial conditions decreases monotonically as coherence increases. Values of the range of initial conditions for each individual participant and coherence level for each of the models are shown in the Appendix in Figure 4.15, illustrating substantial variability among participants.



Average IC Var Parameters

Figure 4.8: Mean values of the range of initial conditions, by model and coherence condition. Bars indicate standard errors for n = 17 subjects. Note that the pure DDM does not allow variability in initial conditions $(s_x = 0)$.

Finally, we note that estimates of task difficulty are in general agreement across all models. In Figure 4.9 we compare the DDM and LBA estimates of drift. Here the LBA drift values have been reduced by $\frac{1}{2}$, so that a drift of 0 in both models corresponds to no signal (coherence = 0), allowing a direct comparison. The effect of coherence level on the model drift parameters was significant with a large effect size (F(4, 64) = 118.80, p < 0.001, $\eta^2 = 0.76$). The interaction of model and condition type is also significant, but the corresponding effect size is modest (F(16, 256) = 5.48, p < 0.001, $\eta^2 = 0.11$). The effect of model type on drift is also significant (F(4, 64) = 7.79, p < 0.001, $\eta^2 = 0.10$). Estimates of drift rates for individual participants are shown in the Appendix: Figure 4.16. These are considerably more uniform than the initial condition ranges shown in Figure 4.15.



Average µ Parameters

Figure 4.9: Mean values of the drift rate parameter, μ , averaged over all participants and conditions for each of the models. Bars indicate standard errors for n = 17 subjects. For the LBA model fits, the relative evidence in favor of option $1 \mu_1 - \frac{1}{2}$ is shown.

4.4 Discussion

We compared the DDM and LBA accounts of behavior in an Rmax task. In the context of the DDM, the adjustment of a caution or threshold parameter is known to be integral to describing Rmax behavior [10, 6]. For example, participants either adjust the threshold to best suit each difficulty condition, or they may pick a single threshold level which works well, but not optimally, across blocks of various difficulty levels [6].

We first compared the DDM and LBA accounts of behavior in the context of optimal performance theory. We showed that, while the OPC for the pure DDM is a single, unique curve, the OPCs for the LBA, like those for the extended DDM are non-unique. We also showed that, for a given set of parameters, the best behavior possible in the LBA is at least partially determined by the value of the range of initial conditions. With a nonzero range of initial conditions, uniformly quick responses at near signal detection speed are impossible. Further, when the range of initial conditions in the LBA, A, is set at or near 0, the quality of fits is limited. Nonetheless, allowing A to vary from coherence to coherence allows for significantly better data fits and this parameter variability is consequently critical to the success of the model.

Implications for the identification of an OPC for LBA parameters are illustrated in Figure 4.2: we found that the shape of the OPC depended upon the allowed range of initial conditions. Smaller, more restricted ranges of initial conditions result in flatter OPCs.

We then applied various DDM and LBA accounts of behavior to an Rmax data set. We considered several models: a pure DDM, an extended DDM, and a standard LBA fit in which drift and threshold varied by difficulty condition. We also considered fits to the extended DDM and to the LBA in which the range of initial conditions was allowed to vary from one coherence level to the next. We showed in Table 4.1 and Figure 4.3 that LBA matches to behavior were best when both the range of initial conditions and the thresholds were allowed to vary with coherence. For similarly good matches to behavior, such manipulation of initial conditions was unnecessary in the DDMs we considered. For consistency, we employed the standard LBA parameterization as described in [20, 39] to generally follow that of the extended DDM.

That initial condition variability is critical in the LBA is to be expected, as we found that the ability to achieve short RTs (and corresponding estimated DTs) in the LBA model is directly influenced by the range of initial conditions, A, as shown in Figure 4.2(a).

Our measurements of average fit quality were consistent with the expected importance of variability in initial conditions when accounting for Rmax behavior in the LBA: quality of fit metrics AIC, AIC_c , and BIC were better for the LBA model in which variance in initial conditions was allowed than for the fits in which this variation was held constant. Averaged over all participants, values of the three fit metrics for each model are illustrated in Figure 4.3. In the second LBA fit, participants mostly reduced caution and also reduced the range of initial conditions in higher coherence blocks: see Figures 4.7 and 4.8. Significantly, in the first LBA fit, with constant variance in initial conditions, participants instead modestly increased caution with coherence, as they did in all three DDM fits (compare Figures 4.6 and 4.7). In the extended DDM, however, we found that the additional degree of freedom in model parameterization allowed by varying the range of initial conditions from one condition to the next did not result in significantly improved fit metric scores. This difference in the importance of initial conditions between the models may be due in part to the fact that in the pure DDM, thresholds go to 0 regardless of the initial conditions, whereas in the LBA model the threshold range is constrained by the range of initial conditions.

Critically, then, we have shown that the DDM and LBA with varying ranges of initial conditions provide fundamentally different accounts of mean behavior in Rmax tasks. In the DDMs, increased participant caution accounts for much of the change in behavior as the coherence level increases from 0 to 32%. In the second LBA, participants are shown to instead reduce caution while simultaneously narrowing the range of initial conditions. Consequently, the accumulation distance, as well as the corresponding RT and ER data, remains comparable between the LBA and DDM fits. However, the interpretations of these fits are very different.

A direction for future work is to re-adjust our interpretation of LBA parameters. A more practical interpretation of the range of initial conditions in the LBA model would be to assume they are consciously controlled to some degree, in tandem with threshold. The consideration of an adapted LBA as introduced in Section 4.2.2 may allow for yet better accounts of behavior which are more consistent with explanations provided by DDM fits. We leave for future inquiry further modifications of the LBA.

Our results raise the broad question of model design and selection. We note that while good overall, both the LBA and DDM accounts of behavior themselves are imperfect [10]. Additional modes of inquiry include looking more to distributions of RTs for OPCs in addition to mean behavior. While normalized mean DTs are of use when calculating theories of optimal performance, and using normalized mean DTs allows for a unique OPC to gauge behavior, this nondimensional approach precludes the use of numerous existing toolsets designed to describe distributions of RT.

Additional numerical and theoretical analysis are also of interest. For example, we wish to compare additional models such as the Leaky Competing Accumulator Model as well as theoretically optimal Bayesian accounts of behavior with the models presented in this chapter.

With our fits and analyses, we identified a key difference between the LBA and DDM accounts of behavior. While the best LBA description of Rmax behavior involved a decrease in variability in initial conditions coupled with a decrease in caution as the stimulus coherence varies from low to high coherence, the best DDM descriptions involved an increase in caution. In summary, the DDM and LBA accounts of Rmax task behaviors provide highly descriptive but inconsistent accounts of the behavior of the participants. The Rmax task paradigm provides a fruitful tool for model comparison.

4.5 Appendix

Figures 4.10-4.16 show details of fits to individual participants for the five models, averaged over all trials in sessions 10-13, for each given coherence.

Figures 4.10 and 4.11 show RTs and ERs versus coherence for the experimental data, and as predicted by fits of the five models. Figure 4.13 shows the mean normalized DTs versus ERs in comparison with the unique OPC for the pure

DDM. Finally, Figures 4.12, 4.14, and 4.15 show reward rates (RR), thresholds (z), and initial condition variability $(s_x, \text{ or } A)$ versus coherence for the model fits.

In each of the figures, participants are presented from left to right and top to bottom based on closeness of behavior to that predicted by the OPC for the pure DDM. Closeness of behavior is measured by the squares of the differences between mean normalized DTs and normalized DTs lying on the OPC for each participant. The participant with the smallest score, Subject 39, appears in the upper left corner of each figure, and the participant with the largest score, Subject 17, appears in the lower right corner of each figure.



Figure 4.10: Comparison of data and fits to RT, in ms, by participant and condition. Several participants have relatively long RTs on the hardest tasks (0 and 4% coherence). Optimal behavior would be to respond at or near signal detection speed on these tasks.


Figure 4.11: Comparison of data and fits to ER, by participant and condition. Several participants (e.g. Subjects 22, 30, and 32) performed at worse than chance accuracy in the 0% coherence condition, and all of the models matched this behavior.



Figure 4.12: Comparison of RR data (total rewards earned in Sessions 10-13) by participant and difficulty condition, in the data and the corresponding model predictions. Subject 29 had an unusual response curve and responded most slowly in the relatively easy 32% coherence trials.



Figure 4.13: Model fits to mean normalized DT and ER data for each difficulty condition and participant. Some of the differences between LBA and DDM predictions here can be attributed to differences between the model estimates of nondecision time.



Figure 4.14: Comparison of values of the caution or threshold parameter in the different model fits, by participant and condition. LBA threshold values have been scaled down by a factor of 5000, to fit on the same axis.



Figure 4.15: Mean values of the variability in initial conditions, separated by participant, model, and difficulty condition. On average, variability in initial conditions appears to be greater at low than high coherences. The LBA values (of A) have been scaled down by a factor of 5000, to fit on the same axis. The pure DDM has zero variability in initial conditions.



Figure 4.16: Mean values of the drift parameter, μ , separated by participant, model, and coherence condition. As expected, drift magnitude increases monotonically with coherence. Note that LBA drift rate estimates have not been shifted, as was done for the subject-averaged data of Figure 4.9.

Chapter 5

Conclusion and Future Directions

In this thesis, we considered the dynamics of human performance in simple classification tasks. Motivated by the potential to better understand and support a human operator, we studied human performance in several simple tasks. We reanalyzed data from two previously published studies and performed one original experiment to find new patterns in sequential decision tasks, and we developed models to account for these patterns. Our primary findings are that:

• Sequential effects can be explained in part by post-error slowing.

We reconsidered prior experiments and models of sequential effects and showed that in each of these experiments there was a significant post-error slowing effect for which the existing models of sequential effects could not account. By incorporating post-error adjustments to the threshold or caution parameter in our model, we were able to generate superior fits to both post-error responses and to the sequential response dynamics. This analysis is described in Chapter 2. • Sequential effects are influenced by the relative likelihood of repetitions and alternations.

We analyzed data from an original study in which the likelihood of a given stimulus being a repetition or alternation of the previous stimulus varied between blocks of trials. We showed that the influence of a repetition or alternation trial on behavior on that trial had strong effects on both RT and ER. This work is described in Chapter 3.

• Reward maximization task performance allows us to differentiate subjects and discriminate between models.

We reanalyzed data from a previous study [6] in which individuals earned a fixed reward for each correct response and were instructed to optimize the rate at which they would earn such rewards, and we compared fits of the pure and extended DDMs as well as the LBA to subject performance on this task. As in that study, we found that a portion of the subjects performed nearly optimally. While average subjects could be modeled adequately by either version of the DDM and the LBA, we found that the two models provide fundamentally different accounts of behavior in Rmax tasks, with the DDM predicting increases in caution and the second version of the LBA model predicting relative decreases in caution. This work is described in Chapter 4.

5.1 Discussion

Our conclusions are the result of a careful analysis of three behavioral experiments. The first experiment tasked subjects to classify a binary stimulus. Previously presented in [28], the task required subjects to identify which one of two types of stimuli appear on a computer screen. The second task required motion discrimination, using the moving dots paradigm with stimuli biased to show more repetitions or alternations in each block of trials, in addition to the usual equally probable case. We have previously presented the results of the second experiment in [52]. The final experiment, reanalyzed from [6], again used equally probable as opposed to biased random stimuli, also with the moving dots paradigm, but with task difficulty levels varied by block and rewards provided for correct responses, so that optimally performing subjects would need to tradeoff speed for accuracy in order to maximize their reward rates.

Each analysis relied heavily upon the manipulation of a response threshold, or caution parameter. In particular, we first considered in Chapters 2 and 3 reactive, sequential adjustments to the threshold, with higher values after an incorrect response and lowered values after a correct one. We then considered in Chapter 4 strategic overall adjustment of thresholds and ranges of initial conditions as difficulty changes from one block of trials to the next. In each case our analysis allowed us to identify differences between the predictions of various models and to suggest which of a group of candidate models would be best suited to account for subject behavior in a given data set.

Most critically, we identified new and powerful ways to discriminate between models of simple two alternative forced choice RT tasks by studying predictions of responses to sequences of stimuli and to rewarded tasks. In particular, in the second and third chapters of this thesis, we considered combinations of factors including post-error slowing, sequential effects in tasks with equally probable stimuli, sequential effects for a task with a bias towards repetitions or alternations, and sequential effects for correct and error trials. In the fourth chapter, we considered reward maximization behavior at various task difficulty levels.

5.2 Future Work

In this thesis we considered separately the manipulation of stimulus sequence and task difficulty; one interesting direction for future work is to study the interaction of sequential effects and task difficulty. Our first experiment served as a control, in which stimulus sequence and task difficulty were held constant. We then modulated the stimulus sequences in our second experiment so that repetitions or alternations would be more likely. However, the interaction of stimulus sequence and task difficulty and the combined influence of these factors on subject performance is of interest for several reasons. Mechanistically, we expect these effects to be modelled by a combination of reactive sequential threshold adjustments, and possibly adjustments of other parameters, as described in Chapter 2, and strategic overall mean threshold adjustments, as described in Chapter 4. Practically, we expect interesting and nuanced interactions here, involving different time scales, because sequential effects on their own have been shown to directly relate to sequence learning [115]. In addition, learning processes and behavior in the world at large are understood to be driven by rewards and punishments [121, 45].

Specifically, we are interested in several additional directions which follow logically from the work presented in this thesis. In particular, we would like to further extend the results of the study of sequential effects in Chapters 2 and 3 by conducting and analyzing the results of a larger scale experiment with a range of task difficulties, allowing us to study the resulting variability in processing times and patterns for correct and incorrect responses. In addition, we would like to consider and compare additional methods of model parameter adjustments, particularly involving variation of the threshold parameter. In this work, we fit models in which the threshold was either fixed or was allowed to increase or decrease by a fixed, finite increment on each trial. Multiple studies have prescribed a trial-to-trial variability in the value of the threshold or allow the threshold to be randomly selected from the distribution [93, 95]. However, to our knowledge, no prior studies have systematically considered and compared alternative methods for threshold adjustments (for example, gradually changing the rate of change of the threshold from trial to trial, or "resetting" the threshold after a certain number of trials). We would like to consider alternate methods of varying the response threshold from trial to trial.

Finally, model comparison methods discussed in this thesis can be used to differentiate between additional models not considered in this thesis, including nonlinear models, such as the Leaky Competing Accumulator. In the context of competing accumulator models, the different effects of varying initial condition versus thresholds can be more carefully considered using methods discussed in this paper.

We are also interested in exploring potential applications of this knowledge to assist the human in human-in-the-loop tasks. For example, in tasks such as vehicle navigation or various security monitoring activities, participants are expected to be fully awake and aware, but are subject to fatigue and distraction. Patterns in the reaction time of an individual, coupled with an estimate of his or her general accuracy, could allow for the early identification and possible prevention of costly errors. Such future projects can help validate our findings and more importantly, they can provide deeper insight into the several areas of study covered by this work, especially engineering and neuroscience.

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