Formation shape and orientation control using projected collinear tensegrity structures

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Abstract— The goal of this work is to stabilize the shape and orientation of formations of N identical and fully actuated agents, each governed by double-integrator dynamics. Using stability and rigidity properties inherent to tensegrity structures, we first design a tensegrity-based, globally exponentially stable control law in one dimension. This stabilizes given inter-agent spacing along the line, thereby enabling shape control of one-dimensional formations. We then couple one-dimensional control laws along independent orthogonal axes to design a distributed control law capable of stabilizing arbitrary shapes and orientations in n dimensions. We also present two methods for formation shape and orientation change, one using smooth parameter variations of the control law, and the other, an n-step collision-free algorithm for shape change between any two formations in n-dimensional space.

I. INTRODUCTION

Formation control strategies are crucial to the performance of multi-agent platforms, such as clusters of satellites, groups of autonomous underwater vehicles (AUVs) and formations of unmanned air vehicles (UAVs). Problems in cooperative formation control often involve maintaining the shape and/or orientation of a formation. We are particularly motivated, for instance, by multi-agent systems serving as mobile sensor networks, as in a recent adaptive sampling field experiment involving a fleet of autonomous underwater vehicles [?]. Depending on the nature of the observations made by a given multi-agent sensing system, certain shapes and orientations of the formation might prove to be particularly advantageous for performance and efficiency of data gathering, data processing and forecasting. For example, Zhang and Leonard [?] presented an algorithm in which the shape of the formation is chosen to minimize the mean least squared error in gradient estimates of the scalar field observed.

In this work, we define the *shape* of a formation of agents by consistent Euclidian distance constraints between every pair of agents in the formation. Hence a formation shape in n-dimensional space ($\mathbb{R}^n$) is invariant under transformations of the Euclidian group $E(n)$ (rotations, translations and reflections). For a constant shape, the formation *orientation* is defined as the orientation of a frame fixed to the formation. From rigidity graph theory [?], it is known that fixing the shape of a formation of N agents requires fewer than

$N(N-1)/2$ pairwise distance constraints. For example, for generic shapes in $\mathbb{R}^2$, $2N-2$ well distributed distance constraints are sufficient [?].

In work by Nabet and Leonard [?], [?], a constructive method is given to stabilize a planar shape for a group of N vehicles using virtual *tensegrity structures*, where each vehicle is modeled as a holonomic agent with double-integrator dynamics. Tensegrity structures ([?], [?]) are geometric structures formed by a combination of struts (in compression) and cables (in tension), which meet at nodes. These structures have been studied extensively in the mathematics literature, where stability and rigidity properties have been proven (e.g. [?], [?]). Hence, by modeling vehicle formations using tensegrity structures (replacing nodes by agents and cables/struts by virtual tensile/compressive spring forces), multi-agent groups can inherit these favorable properties.

Here, we first study the simple case of formation shapes in one dimension and, using ideas from [?], [?], design a globally exponentially stable control law to stabilize these shapes. We then employ a projection of one-dimensional controllers to stabilize the shape and orientation of formations in n-dimensional space. Note that in [?], [?], the formation control is distributed (agents measure relative positions of designated neighbors) and allows for stabilization of the formation shape regardless of orientation. In the present work, we require also that agents have knowledge of the orientation of a common formation reference frame; this gives us the added ability to prescribe specific formation orientations.

The formation control law proposed here yields global exponential stabilization to the desired formation shape and orientation. The distributed control law requires a maximum nN undirected communication links between agents; the linear scaling in N is particularly advantageous for formations of large numbers of agents. Smooth parameter variations in the control law yield smooth skewing or rotation of the formation shape. We also design a collision-free algorithm for shape change between arbitrary n-dimensional formation shapes.

In Section II we consider formation stabilization in one dimension and generalize to formations in $\mathbb{R}^n$ in Section III. We specialize to planar formations in Section IV and in Section V we consider formation shape changes. Section VI comprises a simulation example and we conclude in Section VII.

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II. ONE-DIMENSIONAL TENSEGRITY-BASED FORMATIONS

The goal of this section is to present a globally exponentially stable control law that stabilizes the shape of a formation of agents along a line, with each agent modeled as a point-mass with double integrator dynamics. Consider a collinear formation of N agents with each agent having position coordinate ζ_i, velocity ξ_i, and control input (force) u_i. Assuming that the agents are identical and of unit mass, the dynamics for each agent, for $i = 1, \cdots, N$, are given by

$$
\begin{aligned}
\dot{\zeta}_i &= \xi_i \\
\dot{\xi}_i &= u_i.
\end{aligned}
\tag{1}
$$

Define relative position coordinates $x_i = \zeta_{i+1} - \zeta_i$, for $i = 1, \cdots, N-1$. The desired shape of the formation is prescribed rigidly by specifying $N-1$ relative position constraints:

$$
x_i = x_{ie}, \; i = 1, \cdots, N-1,
$$

where $x_{ie} > \delta > 0$ for all i. We define $X = \sum_{i=1}^{N-1} x_{ie}$ as the distance between the two terminal agents of the formation, as illustrated in Figure 1.
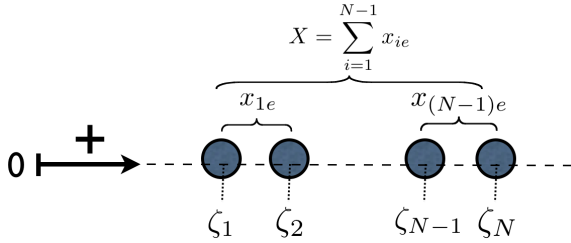

Fig. 1. Collinear formation of N agents at desired shape.

Following the work in [?], [?], the interaction between agents is modeled in the setting of tensegrity structures by designing virtual springs of finite rest length, to represent the tensegrity struts and cables. Springs representing cables (struts) are designed to have a rest length shorter (correspondingly longer) than the desired equilibrium distance between the agents they connect. For clarity of analysis, following the prescription in [?], we define the rest length for cables (struts) to be one-half (double) the desired equilibrium distance between the agents they connect. For two agents labeled i and j connected by a spring of spring constant $k_{ij} = k_{ji}$ and rest length $l_{ij} = -l_{ji}$, the force exerted by agent i on agent j (represented as $F_{i \to j}$) is given by

$$
F_{i \to j} = -k_{ij}(\zeta_j - \zeta_i - l_{ij}) = -F_{j \to i}.
\tag{2}
$$

Note that unlike in [?], [?], the springs defined here have directionality. The force acting on a specific agent (u_i in (1)) is given by the cumulative effect of the spring forces exerted by the agents to which it is connected, i.e.,

$$
u_i = -\nu \xi_i + \sum_{j \in \mathcal{N}_i} F_{j \to i},
\tag{3}
$$

where $\mathcal{N}_i$ is a set of indices corresponding to those agents with a virtual spring interconnection to agent i ($k_{ij} \neq 0$, $\forall j \in \mathcal{N}_i$), and $\nu > 0$ is a damping coefficient.

Fig. 2. Shape coordinates along the line. The dashed lines represent *cable* interconnections and the solid line represents a *strut*.

We assign successive ordered pairs of agents to be linked by cables and the two terminal agents to be linked by a strut, as shown in Figure 2. Spring constants and rest lengths for the N springs are defined as follows, where $k > 0$ is a constant and $(i, j) = (1, 2), (2, 3), \cdots, (N-1, N), (1, N)$ are the index pairs of linked agents:

$$
\begin{aligned}
l_{ij} = -l_{ji} &= \begin{cases} \frac{x_{ie}}{2} & \text{if } j = i+1 \\ 2X & \text{if } i = 1 \text{ and } j = N \end{cases} \\
k_{ij} = k_{ji} &= \begin{cases} \frac{2kX}{x_{ie}} & \text{if } j = i+1 \\ k & \text{if } i = 1 \text{ and } j = N. \end{cases}
\end{aligned}
\tag{4}
$$

For a set of $N \geq 3$ agents, the choice of spring connections prescribed in (4) stabilizes the desired formation shape and results in an undirected ring communication topology between the agents with N communication links. To show this we define a Lyapunov function $E = T + V$ as the total energy of the system, where, V is the potential energy and T is the kinetic energy given by

$$
V = \frac{1}{2}k \left(\sum_{i=1}^{N-1} x_i - 2X \right)^2 + \sum_{i=1}^{N-1} \frac{kX}{x_{ie}} \left(x_i - \frac{x_{ie}}{2} \right)^2
$$
$$
T = \frac{1}{2} \sum_{i=1}^{N-1} \xi_i^2.
\tag{5}
$$

Note that the potential function V is consistent with the definition of the forces in (2), i.e.,

$$
\sum_{j \in \mathcal{N}_i} F_{j \to i} = -\frac{\partial V}{\partial \zeta_i}.
$$

Proposition 1: The tensegrity-based control law, defined in (2), (3) and (4), globally exponentially stabilizes the desired one-dimensional formation shape specified by $x_{ie} > \delta > 0$ (Figure 1) for the dynamics (1).

Proof of Proposition 1: E is a positive definite and radially unbounded function with the desired formation shape a global minimum. We compute $\dot{E} = -\nu \sum_{i=1}^{N} \xi_i^2 \leq 0$. By the LaSalle Invariance Principle [?], all solutions converge to the largest invariant set contained in $S_1 = \left\{ \zeta_i, \xi_i \mid \dot{E} = 0 \right\} = \left\{ \zeta_i, \xi_i \mid \xi_i = 0, \frac{\partial V}{\partial \zeta_i} = 0 \right\}$.

We have that $\frac{\partial V}{\partial \zeta_i} = 0$, $\forall i = 1, \cdots, N$, if and only if $\frac{\partial V}{\partial x_i} = 0$, $\forall i = 1, \cdots, N-1$. Define stacked vectors $\vec{x} = \begin{bmatrix} x_1 & \cdots & x_{N-1} \end{bmatrix}^T$ and $\vec{x}_{eq} = \begin{bmatrix} x_{1e} & \cdots & x_{(N-1)e} \end{bmatrix}^T$. Let $\mathbf{1}_{N-1} \in \mathbb{R}^{N-1}$ be the vector of all ones. $\vec{x}$ is a critical point of V if and only if

$$M\vec{x} = B, \tag{6}$$

where $M \in \mathbb{R}^{(N-1) \times (N-1)}$, $B \in \mathbb{R}^{N-1}$,

$$M = \begin{bmatrix} \frac{2X}{x_{1e}} + 1 & 1 & \cdots & 1 \\ 1 & \frac{2X}{x_{2e}} + 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & \frac{2X}{x_{(N-1)e}} + 1 \end{bmatrix} \tag{7}$$

and

$$B = \begin{bmatrix} 3X\mathbf{1}_{N-1} \end{bmatrix}. \tag{8}$$

Since the symmetric matrix M is the sum of a positive definite diagonal matrix and the positive semi-definite matrix of all ones, it is positive definite and therefore invertible. One can compute explicitly that $\vec{x}_{eq} = M^{-1}B \in \mathbb{R}^{N-1}$ is the unique solution to (6). Convergence to the desired shape is exponential since the dynamics are linear. ∎

The next corollary, used in Section V, follows from Proposition 1 and the definition of exponential convergence.

Corollary 1: For every $\epsilon > 0$, there exists a $T_\epsilon < \infty$ such that for all $t > T_\epsilon$, $\|\vec{x}(t) - \vec{x}_{eq}\| + \|\vec{\xi}(t)\| < \epsilon$.

Remark 1: Equations (2) and (3) imply that the momentum of the formation is conserved for zero initial momentum, or decays exponentially to zero for nonzero initial momentum.

III. n-DIMENSIONAL TENSEGRITY-BASED FORMATIONS

In this section we present a systematic methodology for global stabilization of formation shapes comprising N agents, each with double integrator dynamics, in n-dimensional space, i.e., $\boldsymbol{\zeta_i} = \begin{bmatrix} \zeta_i^1 & \cdots & \zeta_i^n \end{bmatrix}^T \in \mathbb{R}^n$, $\forall i = 1, 2, \cdots, N$. For a given desired equilibrium shape and orientation, consider coordinates defined by choosing a formation agent as the reference (assume agent $i = 1$) and projecting relative coordinates of all agents onto an orthogonal reference frame (formation frame) fixed to the reference agent and with fixed orientation relative to the inertial frame. Consider the space defined by the coordinates $\mathbf{z_i} \in \mathbb{R}^n$, where, $\mathbf{z_i} = \boldsymbol{\zeta_i} - \boldsymbol{\zeta_1}$, $i = 2, 3, \cdots, N$. We study formation dynamics in this space $\mathbb{R}^{n(N-1)}$ ($\mathbb{R}^{nN}$ modulo translations of the formation as whole). In order to design a formation shape control algorithm in this space, we first make several assumptions.

Assumption 1: The desired equilibrium formation corresponds to finite distances between all agents, i.e., $\|\mathbf{z_{ie}} - \mathbf{z_{je}}\| > \delta > 0$ $\forall i \neq j$ (note that the subscript e represents the equilibrium values of these coordinates).

Assumption 2: Every agent has access to the fixed orientation of the same unique formation frame and is thus able to express its measurements of relative positions to other agents in a common reference frame.

Assumption 3: Agents are holonomic, i.e. fully actuated.

The condition for the existence of the formation equilibrium in one dimension is the invertibility of matrix M in (6), which holds if the equilibrium distances between agents are nonzero. The analogous condition in n dimensions is that there should be nonzero distances between the projections of the agents' equilibrium positions along the n orthogonal axes of the formation frame. The existence of *nonsingular* frames that make this possible for arbitrary formations in n dimensions is proved in Lemma 1.

Lemma 1: For any formation equilibrium in n dimensions that satisfies Assumption 1, there exists a non-empty set of orthogonal n-dimensional formation frames, each of which yields nonzero distances between the projections of agent equilibrium positions along the n orthogonal axes, i.e. $z_{ie}^k \neq z_{je}^k$, $\forall k \in \{1, 2, \cdots, n\}$ and $i, j \in \{2, \cdots, N\}$, $i \neq j$.

Proof of Lemma 1: Consider selecting agent labeled $i = 1$ as the reference agent and prescribing a formation frame centered at that agent. There are no more than a finite number of singular formation frames centered at agent $i = 1$ that correspond to two or more agents having the same coordinates on some frame axis, for finite number of agents N. Hence the set of frames *not* satisfying the nonzero distances property along axes is non-empty. ∎

The proof of Lemma 1 implies that we have abundant freedom in selecting nonsingular formation frames as long as we avoid the finite set of singular frames. Note however that in practice, finding such a frame becomes more challenging for formation shapes with closely spaced agents. For any nonsingular frame, Proposition 1 gives a one-dimensional formation shape control law corresponding to projections of agent positions along each axis of the frame. The composition of one-dimensional control laws along mutually orthogonal axes implies that for each pair of agents i and j, $(z_i^k - z_j^k) \to (z_{ie}^k - z_{je}^k)$ at equilibrium, $\forall k \in \{1, 2, \cdots, n\}$, which further implies $\mathbf{z_i} - \mathbf{z_j} \to \mathbf{z_{ie}} - \mathbf{z_{je}}$. This globally exponentially stabilizes the desired shape of the formation, as summarized in Theorem 1.

Theorem 1: Consider a formation of agents in $\mathbb{R}^n$ satisfying Assumptions 1-3 and with equilibrium coordinates $\mathbf{z_{ie}} \in \mathbb{R}^n$, $i \in \{2, 3, \cdots, N\}$, projected along a nonsingular orthogonal formation frame. The formation control law comprising a composition of one-dimensional shape controllers as defined by (2), (3), (4), along independent frame axes, globally exponentially stabilizes the n-dimensional formation shape.

Remark 2: The convergence for formation dynamics in n dimensions to a shape and orientation, as described in Theorem 1, is global and exponential analogous to Proposition 1. Also, the control requires a maximum of nN communication links.

IV. SPECIALIZATION TO PLANAR DYNAMICS

In this section we consider the $n = 2$ case of the formation control described in Section III. This case is of particular interest because it corresponds to studying planar formations of vehicles modeled as point masses. In order to apply the formation control law from Section III, Assumptions

1-3 must be satisfied. Assumption 2 is akin to equipping agents with a magnetic compass and programing them with a reference compass heading corresponding to the orientation of the formation frame.

Designing the formation control law requires the choice of a nonsingular formation reference frame, the existence of which is proved in Lemma 1. As an example, the set of possible frame choices for a square-shaped formation is illustrated in Figure 3. In general, for arbitrary planar formations, the choice of formation frame can be made specific by considering a *best-spacing* condition. Specifically, we see from (4) that axes projections with well spaced agents ($\min(X/x_{ie}) \ \forall i$) result in dynamics with lower gains (spring constants k_{ij}) on individual springs. One can solve an optimization problem over the set of reference frames to obtain the frame that corresponds to optimal spacing along both formation frame axes. The solution to such a problem for a square-shaped formation is illustrated in Figure 3.
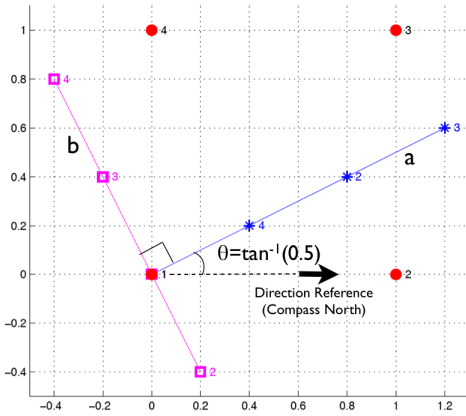

Fig. 3. A planar square formation (agent positions are circles) showing choice of formation frame parameterized by heading θ. Note that $\theta \in (0, \frac{\pi}{4})$ parameterizes a set of nonsingular frames. It can be shown $\theta = \tan^{-1}(.5)$ corresponds to optimal spacing along the axes. Labeled stars and squares show agent positions projected along the $\mathbf{a}$ and $\mathbf{b}$ axes, respectively.

Given a choice of reference frame, we can apply Theorem 1 to design a distributed formation shape control law that stabilizes the desired formation shape. Note that different permutations of reflections of the formation frame axes can specify various formation reflections and orientations, each having the same Euclidian distances between agents. The choice of frame isolates one of these four possibilities in the plane as illustrated for a generic asymmetrical formation of $N = 5$ agents in Figure 4. The communication topology corresponding to the formation in Figure 4 is presented in Figure 5. According to Remark 2, no more that $2N = 10$ links are required; here only 8 links are used. This is equivalent to the $2N - 2 = 8$ distance constraints sufficient for fixing a generic planar shape, following rigidity graph theory [?].

V. FORMATION SHAPE CHANGES

In this section we study two ways to reconfigure the shape and orientation of a formation.
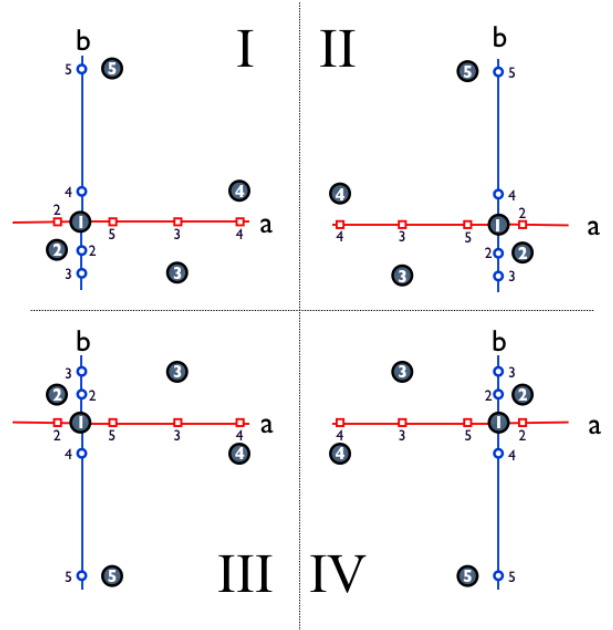

Fig. 4. Illustration of four possible equilibrium configurations in the plane for an arbitrary five-agent formation shape and the same frame orientation; each configuration is distinguished by a different choice of axes directions. The configurations all have the same relative spacing of agent positions projected onto the formation frame (labeled $\mathbf{a} - \mathbf{b}$). The labeled squares and circles along the $\mathbf{a}$ and $\mathbf{b}$ axes are agent coordinate projections along those axes. Note that all equilibria are identical modulo $E(2)$ transformations and further pairs (I, IV) and (II, III) are identical modulo an $SO(2)$ transformation.
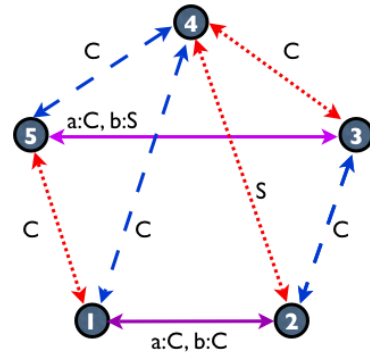

Fig. 5. Communication topology corresponding to the five-agent formation in Figure 4. An undirected link between two agents represented by an arrow, indicates that each agent is able to measure the position of the other agent relative to itself. The dotted (dashed, solid) links correspond to measurements sufficient for stabilization along the $\mathbf{a}$-axis ($\mathbf{b}$ axis, both axes). Links are labeled C and S to indicate cable and strut, respectively.

A. Smooth parameter variations

Here we apply results from Chapter 9.6 of [?], particularly the fact that close to an isolated exponentially stable equilibrium point of a system, dynamics caused by changes in initial conditions are much faster than those caused by changes in slowly time-varying parameters of the system. This approach is also used by Nabet and Leonard [?] for shape changes of planar formations. For formation shape change we consider the formation frame orientation and the formation spring

rest lengths l_{ij} along the axes as the key parameters for variation. Theorem 9.3 from [**?**] states that as long as the parameter variations are smooth in time and slow enough[1], uniform boundedness of trajectories and well-behavedness of solutions is ensured. Thus we can stably change the shape of the formation by navigating through a parameterized space of formation equilibria.

For example, consider the controlled dynamics of an N-agent planar system initially at rest near the equilibrium shape, where control is as defined in Theorem 1. By smoothly rotating the axes of the formation (changing θ in Figure 3) we can reorient the formation. By smoothly scaling the spring rest lengths l_{ij} along each axis, we can skew the shape of the formation. Figure 6 illustrates these parameterized shape changes for a planar square formation. The formation skewing and rotation can be done simultaneously and in a periodic fashion to produce dynamically time-varying formation shapes, well suited for certain adaptive sampling problems [**?**] (Figure 6D).
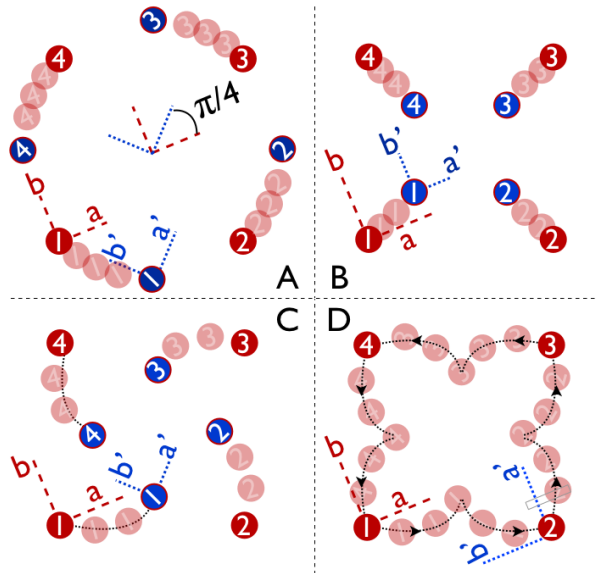

Fig. 6. Parameterized formation shape changes for a planar square formation initially at rest. Red circles and corresponding axes $\mathbf{a} - \mathbf{b}$ indicate initial formation agent positions and reference frame orientation; blue circles and corresponding axes $\mathbf{a}' - \mathbf{b}'$ indicate agent positions and frame orientation after shape change. A) Rotation of formation about center of mass by $45°$. B) Scaling size of formation to one-half the original. C) Scaling formation to one-half size while rotating it by $45°$. D) Periodically time-varying, synchronized scaling and rotation for a quarter cycle ($90°$ rotation); notice that agents interchange their location on the plane in the process.

B. Collision-free reconfiguration

The formation control described in Theorem 1 is decoupled along the n axes of the formation frame. This decoupling and exponential convergence along independent

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axes is convenient to an n-step algorithm to change from one arbitrary formation shape to another without collisions.

Consider two n-dimensional formation shapes labeled Ψ and Ω, and each projected along the same formation frame centered at a given agent and comprising n orthogonal vector directions labeled $\{\boldsymbol{\phi_i}\}_{i=1}^n$. Corollary 1 states that formation convergence along frame axes to an arbitrarily small ϵ-neighborhood of desired shape is finite-time. This yields the following n-step algorithm for shape change from Ψ to Ω.

Step 1: Start from formation Ψ initially at rest and reset the control virtual spring parameters along the $\boldsymbol{\phi_1}$ axis to corresponding parameters for formation Ω. Here the formation eventually stabilizes to an intermediate formation denoted $\Omega(\boldsymbol{\phi_1}), \Psi(\boldsymbol{\phi_2}), \cdots, \Psi(\boldsymbol{\phi_n})$, where $\Omega(\boldsymbol{\phi_i})$ ($\Psi(\boldsymbol{\phi_i})$) indicates that coordinates projected on the $\boldsymbol{\phi_i}$ axis correspond to formation Ω (Ψ).

Steps 2 to n: After waiting for a time period T_i for the convergence of the previous step i, progressively reset the control virtual spring parameters to corresponding parameters for formation Ω, independently for each axis of the formation frame, in order to stabilize to the final formation with shape given by Ω.

Corollary 2: There exists an $\epsilon > 0$ such that for $T_i > T_\epsilon(\boldsymbol{\phi_i})$, where $T_\epsilon(\boldsymbol{\phi_i})$ is the convergence time for stabilization along each formation frame axis $\boldsymbol{\phi_i}$ (according to Corollary 1), the n-step shape change algorithm for planar formations above is collision-free.

For a collision to occur during the formation shape change, a given pair of agents *must* have the same projection coordinates along *each* of the formation axes. The decoupled nature of the control and the n-step prescription above are hence collision-free because during each step, a finite spacing between agents is ensured as projections along $n - 1$ axes remain stable. This observation points to a simpler procedure involving just two steps for collision-free formation shape change. The first step involves holding control parameters along any one axis (say $\boldsymbol{\phi_k}$) constant, and resetting the parameters along all other axes to those of the final formation. After stabilization of the intermediate formation, the second step comprises resetting the control parameters corresponding to the $\boldsymbol{\phi_k}$ axis to that of the final formation.

VI. ILLUSTRATIVE SIMULATION

Here we simulate the planar formation shape stabilization dynamics described in Section IV by employing the control law defined in Theorem 1, and a shape change maneuver as described in Section V(B). Consider a formation of four agents each initially at rest and with arbitrary initial positions:

$$\boldsymbol{\zeta^1}(0) = \begin{bmatrix} 0.4 & 0 & -0.8 & 0.4 \end{bmatrix}^T$$
$$\boldsymbol{\zeta^2}(0) = \begin{bmatrix} 1 & -0.1 & 0 & -0.9 \end{bmatrix}^T. \tag{9}$$

We simulate stabilization followed by shape change as illustrated in Figure 7. First we stabilize the formation to a square of side one unit in length. Next, we perform a two-step collision-free shape change maneuver to transform the

formation from a square to an equilateral triangle of side one unit with an agent at its centroid. For this simulation we choose $\nu = 1$, $k = 0.1$ for each axis, and formation frame such that $\theta = \pi/12$ throughout (see Figure 3 for definition of θ). Notice that in Figure 7 the formation shape change is indeed collision-free.
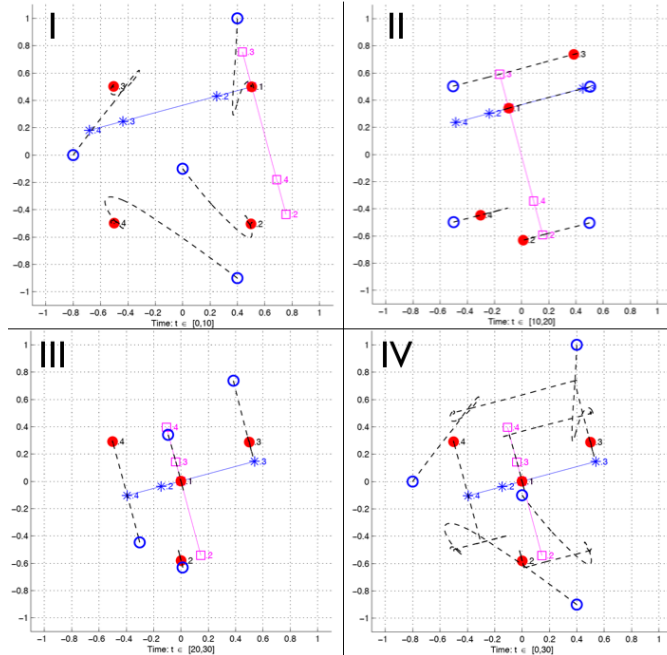

Fig. 7. Simulation of formation dynamics and shape change. In each plot, the hollow (filled) circles represent positions at the beginning (end) of the time period corresponding to that plot, dashed lines are trajectories. Axes projections at the end of each period are depicted as labeled stars and squares. The formation frame (stars and squares) is inclined at an angle of $\pi/12$ to the reference frame (background grid). I) From initial condition to $t = 10$, square stabilization. II) From $t = 10$ to $t = 20$, stabilization of Step 1 of the two-step shape change procedure. III) From $t = 20$ to $t = 30$, stabilization of final triangular formation. IV) Composite plot from $t = 0$ to $t = 30$.

VII. CONCLUSIONS

In this paper, we study the control of the shape and orientation of formations of holonomic mobile agents in the setting of tensegrity structures. Our first result is a globally exponentially stable control law for one-dimensional formation shapes, based on tensile and compressive tensegrity spring forces acting synergistically on the various agents of the formation. This one-dimensional formation control law is used for shape control of formations in higher dimensions by controlling projections of formation shapes along orthogonal axes. Formation shape change is studied using smooth parameter variations of the control law, and a collision-free n-step algorithm for arbitrary formation shape change is presented.